



清华大学 未央书院

Weiyang College, Tsinghua University

班级:

姓名: 陈鲲鹏

编号: 2020012957

第 页

4.3

$$4.1) \text{ 令 } g(x) = \frac{1}{1-x+x^2} = \frac{1}{1-(x-x^2)}$$

$$g(x) = 1 + (x-x^2) + (x-x^2)^2 + (x-x^2)^3 + (x-x^2)^4 + o[(x-x^2)^4], \quad (x-x^2) \rightarrow 0$$

$$= 1 + x - x^3 - x^4 + o(x^4)$$

$$\text{原式} = g + xg + x^2g = 1 + 2x + 2x^2 - 2x^4 + o(x^4), \quad \text{即 } P_4 = 1 + 2x + 2x^2 - 2x^4$$

$$(b) y = \frac{x-2}{[(x-2)+2][(x-2)-2]} = \frac{1}{2} \left(\frac{1}{t+2} + \frac{1}{t-2} \right) = \frac{1}{4} \left(\frac{1}{1+\frac{t}{2}} + \frac{1}{\frac{t}{2}-1} \right), \quad t = x-2 \rightarrow 0$$

$$\therefore y = -\frac{1}{2} \left(\frac{1}{2} + \left(\frac{t}{2}\right)^3 + \left(\frac{t}{2}\right)^5 + \dots + \left(\frac{t}{2}\right)^{2n-1} \right) + o(t^{2n}), \quad t \rightarrow 0$$

$$= -\frac{(x-2)}{4} - \frac{(x-2)^3}{2^4} - \frac{(x-2)^5}{2^6} - \dots - \frac{(x-2)^{2n-1}}{2^{2n}} + o[(x-2)^{2n}]$$

$$(8) y = \frac{1}{2} [\ln(1+x) - \ln(1-x)]$$

$$\therefore y_n = \begin{cases} x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^n}{n}, & (n \text{ 为奇数}) \\ x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{n-1}}{n-1}, & (n \text{ 为偶数}) \end{cases}$$

$$(9) \text{ 令 } g(u) = e^{-\frac{1}{u}} \quad (u > 0), \quad g^{(n)}(u) = \sum_{k=1}^n \frac{k! e^{-\frac{1}{u}}}{u^{k+1}}$$

$$\forall m \in \mathbb{N}_+, \quad \lim_{u \rightarrow 0^+} \frac{e^{-\frac{1}{u}}}{u^m} = 0, \quad \text{即 } \lim_{u \rightarrow 0^+} g^{(n)}(u) = 0$$

$$\text{故 } y^{(n)} = 0, \quad P_n = 0$$

$$5.1) \text{ 令 } t = \frac{1}{x}, \quad x - x^2 \ln(1+x) = \frac{t - \ln(1+t)}{t^2} = \frac{\frac{t^2}{2} + o(t^2)}{t^2}, \quad t \rightarrow 0$$

$$\text{故原式} = \lim_{t \rightarrow 0} \frac{t - \ln(1+t)}{t^2} = \lim_{t \rightarrow 0} \frac{\frac{t^2}{2} + o(t^2)}{t^2} = \frac{1}{2}$$

$$(2) \frac{\frac{x^2}{2} + 1 - \sqrt{1+x^2}}{(\cos x - e^{x^2}) \sinh x^2} = \frac{1 + \frac{x^2}{2} - 1 - \frac{x^2}{2} + \frac{x^4}{8} + o(x^4)}{(1 - \frac{x^2}{2} - 1 - x^2 + o(x^2))(x^2 + o(x^4))} = \frac{\frac{x^4}{8} + o(x^4)}{-\frac{3}{2}x^4 + o(x^4)}, \quad x \rightarrow 0, \quad \text{故原式} = -\frac{1}{12}$$

$$7.12) \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4(1+\xi)^4} \quad \xi \in (0, x)$$

$$\text{取 } x = 0.02, \quad x - \frac{x^2}{2} + \frac{x^3}{3} \approx 0.019803, \quad \Delta_1 \leq 5 \times 10^{-7}$$

$$\Delta_2 = \left| \ln(1+x) - x + \frac{x^2}{2} - \frac{x^3}{3} \right| = \frac{x^4}{4(1+\xi)^4} < 10^{-7}$$

$$\therefore \ln(1+x) \approx 0.01980, \quad \Delta < 10^{-5}$$

四章复习题

$$9. \quad \forall x_0 \in [0, 1]$$

$$f(0) = f(x_0) - f'(x_0)x_0 + f''(\xi_1) \cdot \frac{x_0^2}{2}$$

$$f(1) = f(x_0) + f'(x_0)(1-x_0) + f''(\xi_2) \cdot \frac{(1-x_0)^2}{2}$$

$$\therefore f(0) = f(1), \text{ 故有 } f'(x_0) = f''(\xi_1) \frac{x_0^2}{2} - f''(\xi_2) \frac{(1-x_0)^2}{2}$$

$$|f'(x_0)| \leq \left| f''(\xi_1) \frac{x_0^2}{2} \right| + \left| f''(\xi_2) \frac{(1-x_0)^2}{2} \right| \leq M \left(\left| \frac{x_0^2}{2} \right| + \left| \frac{(1-x_0)^2}{2} \right| \right) \leq \frac{M}{2}$$

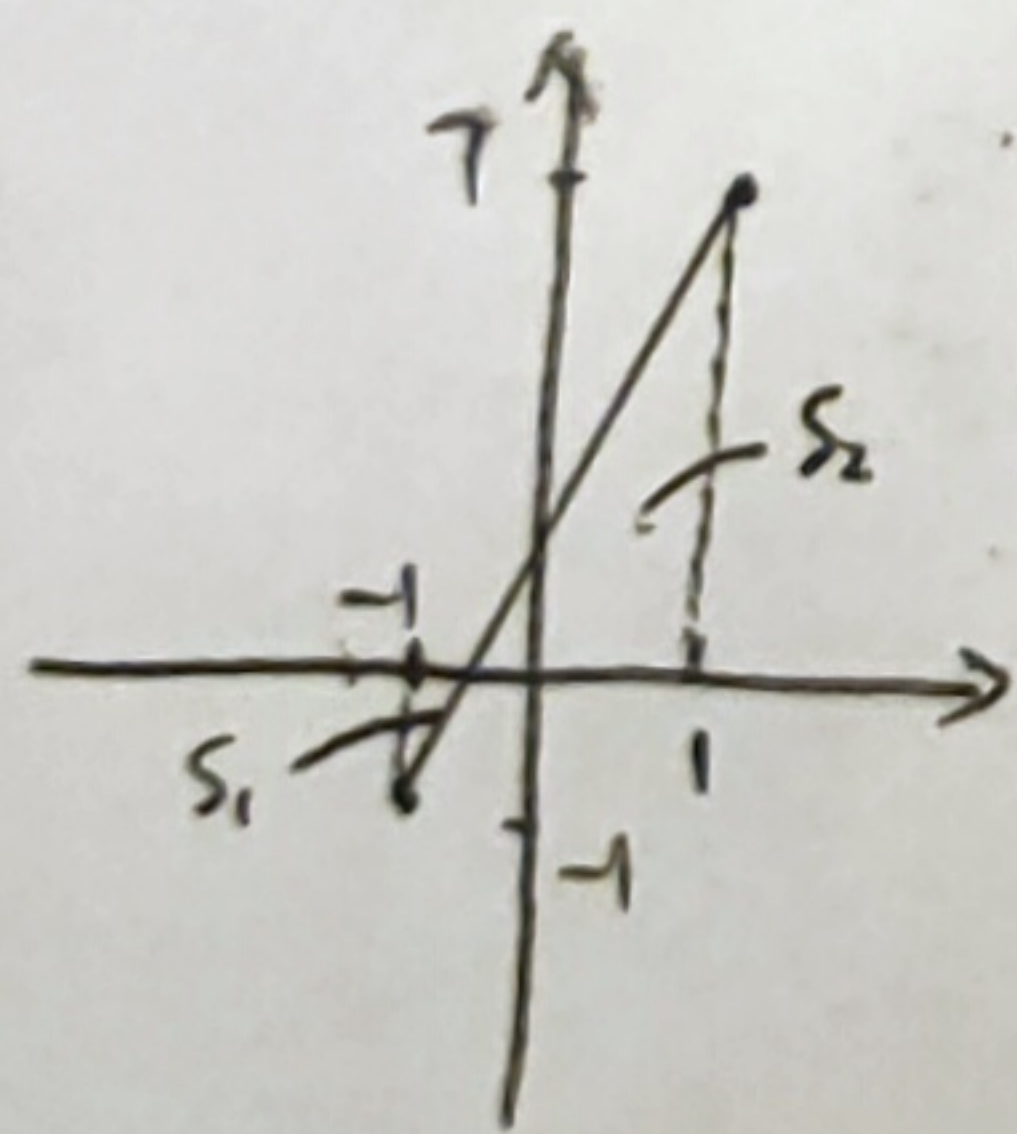
$$11. \quad y \in C[0, 1] \text{ 且 } \min f(x) = -1, \text{ 故 } \exists \eta \in [0, 1], \text{ 使 } f(\eta) = -1, \text{ 且 } f'(\eta) = 0$$

$$f(0) = f(\eta) + \frac{f''(\xi_1)}{2} (0-\eta)^2 \quad f(1) = f(\eta) + \frac{f''(\xi_2)}{2} (1-\eta)^2$$

$$\text{即 } f''(\xi_1) = \frac{2}{\eta^2}, \quad f''(\xi_2) = \frac{2}{(1-\eta)^2}, \quad \text{取 } f''(\xi) = \max\{f''(\xi_1), f''(\xi_2)\}, \text{ 则 } f''(\xi) \geq 8$$

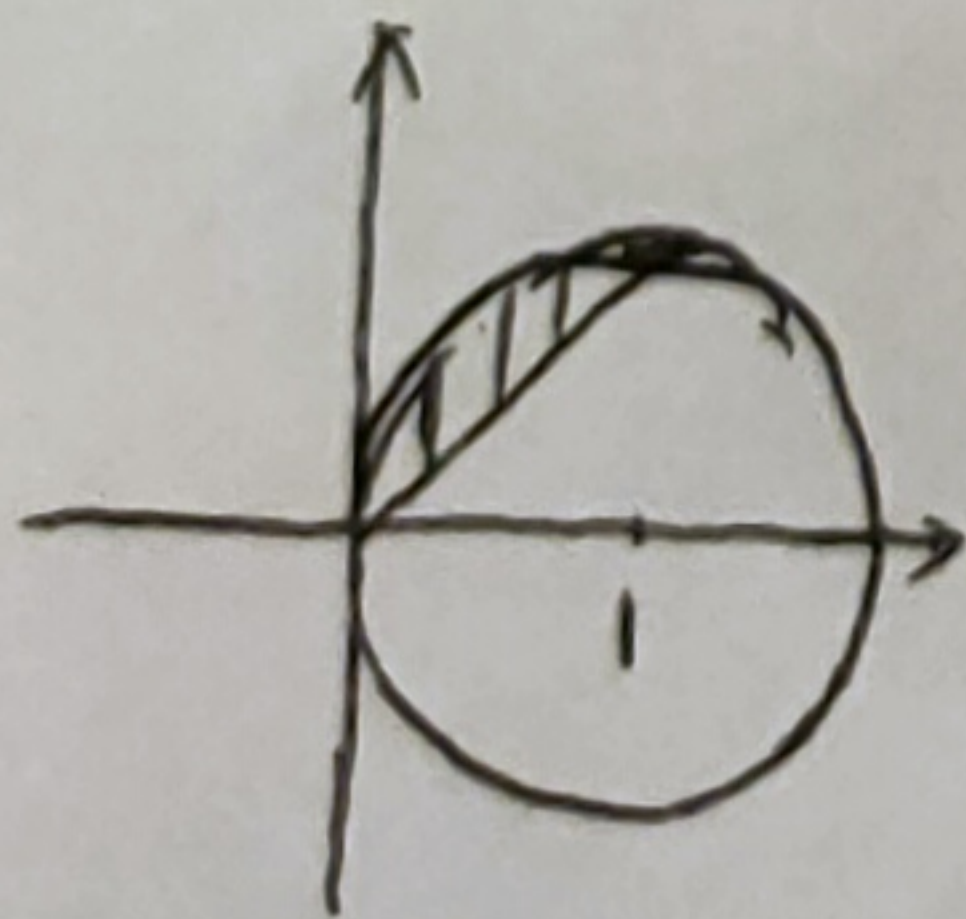
5-1

1. (3)



$$\int_{-1}^1 (4x+3)dx = s_2 - s_1 = 6$$

(4)



$$\int_0^1 (\sqrt{2x-x^2} - x) = \frac{\pi}{4} - \frac{1}{2}$$



清华大学 未央书院

Weiyang College, Tsinghua University

班级:

姓名:

编号:

第 页

5.1

2. 由 $f(x) \leq g(x)$, 知存在 $[a, b]$ 的分割 T , 使得 $U(f, T) \leq L(g, T)$

$$\text{故 } \int_a^b f(x) dx \leq U(f, T) \leq L(g, T) \leq \int_a^b g(x) dx$$

反之不恒成立

4. $f \in R[a, b]$, 故 f 有上界, 又 $f(x) \geq d > 0$, 即 $\frac{1}{f(x)} \in (0, \frac{1}{d}]$ 有界
由 f 定有界, 知 f 与 $\frac{1}{f}$ 间断点相同, 即 $\frac{1}{f}$ 的间断点也是零测度集
综上 $\frac{1}{f(x)} \in R[a, b]$

6. $f \in R[a, b]$, 故 f 有界, 由 F 定义, F 也有界

F 的间断点均为 f 的间断点, 即 F 的间断点集是零测度集

综上 $F(x) \in R[a, b]$

定理 5.1.4

f 在 $[a, b]$ 有界, 不妨令 $|f| < M$. 为方便, 设 f 存在一个断点 $c \in (a, b)$, ~~由断点不连续~~

取 $c_1 > c - \delta$, $c_2 < c + \delta$, $\delta = \frac{\varepsilon}{8M}$ 且 $c_1 < c < c_2$, 则 f 在 $[a, c_1]$ 和 $[c_2, b]$ 连续

故存在分割 $T_1: x_0 < x_1 < \dots < c_1$, $T_2: c_2 < y_1 < y_2 < \dots < b$, 使

$$M_1 = U(f_{[a, c_1]}, T_1) - L(f_{[a, c_1]}, T_1) < \frac{\varepsilon}{4}$$

$$M_2 = U(f_{[c_2, b]}, T_2) - L(f_{[c_2, b]}, T_2) < \frac{\varepsilon}{4}$$

考虑 $[a, b]$ 上分割 $T: x_0 < x_1 < \dots < c_1 < c_2 < y_1 < \dots < b$, 有

$$U(f, T) - L(f, T) \leq M_1 + M_2 + (c_2 - c_1) \cdot [\sup(f(x)) - \inf(f(x))] < \frac{\varepsilon}{4} + \frac{\varepsilon}{4} + 2 \cdot 2M \cdot \frac{\varepsilon}{8M} = \varepsilon$$

故 $f \in R[a, b]$

断点在端点同理, 有多个不连续断点, 总可如上每次多加一个断点, 得证