

Mathematics for Computer Science:

Homework 6

Xiaozhi Wang, 2016011257

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Special Problem 1

Answer: (a):

$$a_n - a_{n-1} = 3(a_{n-1} - a_{n-2}) + 3n + 1$$

Let $b_n = a_n - a_{n-1}$, so $b_n = 3b_{n-1} + 3n + 1$, $b_1 = 1, b_2 = 10$.

$$b_n = 3b_{n-1} + 3n + 1 \quad (1)$$

$$b_{n-1} = 3b_{n-2} + 3n - 2 \quad (2)$$

$$(1) - (2) : b_n - b_{n-1} = 3(b_{n-1} - b_{n-2}) + 3 \quad (3)$$

Let $c_n = b_n - b_{n-1} + \frac{3}{2}$, $c_n = 3c_{n-1}$. $c_2 = \frac{21}{2}$.

So $c_n = \frac{7}{2} \times 3^{n-1}$.

$$b_n = b_1 + \sum_{i=2}^n (c_i - \frac{3}{2}) = 1 + \frac{7(3^n - 3)}{4} - \frac{3(n-1)}{2} = \frac{7 \times 3^n - 11}{4} - \frac{3n}{2}.$$

$$a_n = a_0 + \sum_{i=1}^n b_i = \frac{7(3^{n+1} - 3)}{8} - \frac{11n}{4} - \frac{3n(n+1)}{4} + 1 = \frac{7 \times 3^{n+1} - 6n^2 - 28n - 13}{8}.$$

(b):

$$\frac{1}{a_n} = \frac{1}{a_{n-1}} + 3$$

$$\frac{1}{a_n} = 1 + 3n$$

$$a_n = \frac{1}{3n+1}$$

Special Problem 2

Answer: