

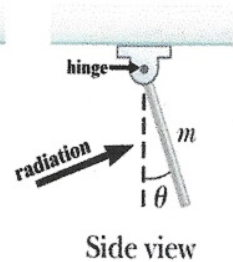
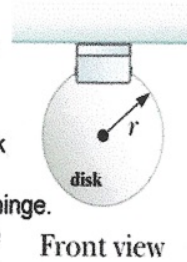
Good Luck !!!

Name: 老毕

Class: \_\_\_\_\_

Student number: SOLUTIONS

1. (15 points) A uniform circular disk of mass  $m = 24.0 \text{ g}$  and radius  $r = 40.0 \text{ cm}$  hangs vertically from a fixed frictionless horizontal hinge (合页) at a point on its circumference (圆周) as shown in the pictures to the right. A beam of electromagnetic radiation with an intensity of  $10.0 \text{ MW/m}^2$  strikes the disk uniformly in a direction perpendicular to the disk's surface. The disk is perfectly absorbing, and the radiation causes the disk to rotate about the hinge. Assuming the radiation always remains perpendicular to the disk's surface as the disk rotates, what will the angle  $\theta$ , as shown in the picture, be when the disk reaches its new equilibrium position? ( $g = 9.8 \text{ m/s}^2$ )



Front view

Side view

The radiation pressure on the disk is  $P = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{S}{c}$  (would have factor of 2 if reflecting)

where  $S$  is the magnitude of the Poynting vector, so  $\frac{\text{energy}}{\text{area} \cdot \text{time}}$

The intensity  $I$  is the time average of  $S$ ,  $I = \langle S \rangle$

but here  $S$  is constant, so  $P = \frac{I}{c} \Rightarrow F = \frac{\pi r^2 I}{c}$

Since the force is constant and uniform over the surface of the disk, just like in a constant gravitational field we can take it to act at the center of the uniform disk.

Calculate torque about hinge:

$\vec{F}_h = F_{hx} \hat{x} + F_{hy} \hat{y}$  make no contribution since there is no lever arm.



$$\sum \vec{\tau}_i = 0 \Rightarrow \frac{\pi r^2 I}{c} r - mg r \sin \theta = 0 \Rightarrow \sin \theta = \frac{\pi r^2 I}{cmg}$$

$$\sin \theta = \frac{\pi (0.400 \text{ m})^2 (10.0 \times 10^6 \text{ W/m}^2)}{(3.0 \times 10^8 \text{ m/s}) (0.0240 \text{ kg}) (9.8 \text{ m/s}^2)} = 0.071238 \Rightarrow \theta = 4.0851^\circ$$

$$\theta = 4.1^\circ$$

2. (15 points) A monochromatic (单色) beam of light of wavelength  $589.0 \text{ nm}$  is used with a Michelson interferometer. The lengths from the beam splitter to the top mirror (which reflects the vertical beam) and the right-side mirror (which reflects the horizontal beam) are fixed to be  $23.0 \text{ cm}$  and  $18.6 \text{ cm}$ , respectively. When a thin piece of fluorite of thickness  $D$  and index of refraction  $n = 1.434$  is inserted into the horizontal beam midway between the beam splitter and the right-side mirror, a shift of 35 fringes (or bright bands) is observed to flow past a pointer on the observation screen for the re-combined beam. What is  $D$ ?

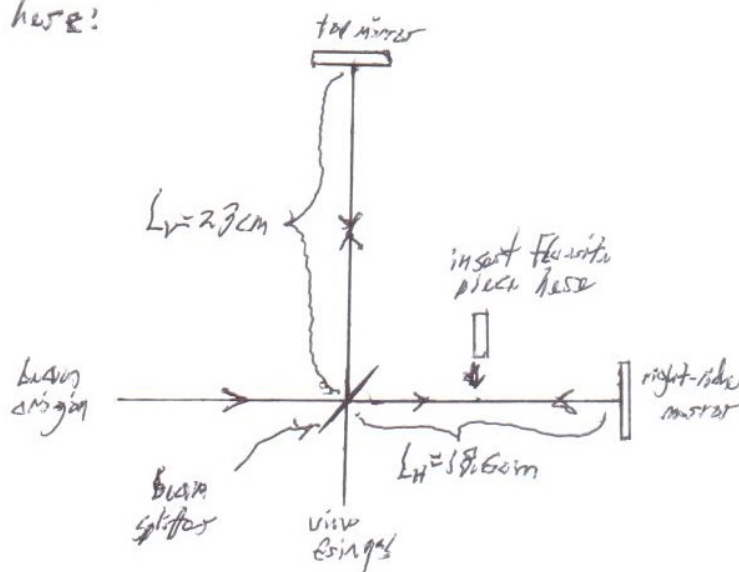
Diagram of Michelson interferometer here:

The optical path difference  
due to the insertion of the  
fluorite piece is

$$2(n_f - n_{\text{air}})D$$

where  $n_f = 1.434$

$n_{\text{air}} = 1.0003 \approx 1$



35 fringes  $\Rightarrow 35\lambda = 2(n_f - n_{\text{air}})D$

$$\therefore D = \frac{35\lambda}{2(n_f - n_{\text{air}})} = \frac{35(589 \times 10^{-9} \text{ m})}{2(1.434 - 1)} = 2.375 \times 10^{-5} \text{ m}$$

$$D = 2.375 \times 10^{-5} \text{ m} = 23.75 \mu\text{m}$$

Note that the numerical values given to you for  $L_v$  and  $L_H$  are not needed. They are "red herrings."

When you are asked to solve problems later in your careers you will not always be told what information is useful and what information is not.

A Seurat painting typically has  $10^9$  dots and took a few years to paint.



3. (14 points) Georges Seurat was a famous neoimpressionist painter of the so-called pointillist school. His paintings are made of a large number of monochromatic (单色) dots each about  $2.50 \text{ mm}$  in diameter and with the space between the dots considerably smaller than the dot size. If the viewer stands far enough away, the colors appear to mix and so how the painting looks depends on from where you look at it! Assuming the typical dot color to be  $550.0 \text{ nm}$ , approximately how far away can you be to resolve the dots (a) with your  $4.00 \text{ mm}$ -diameter eye pupil; and (b) with a  $25.0 \text{ mm}$  camera aperture?

The Rayleigh criterion yields the formula  $\theta_{\text{min}} = 1.22 \frac{\lambda}{D}$  (Eqn 38-19 int ext)  
 where  $D$  = diameter of circular aperture  
 and  $\theta$  is measured in radians or just  $\theta_{\text{min}} \approx \frac{\lambda}{D}$  (Eqn 38-14 int ext)

For small angles,  $\theta \approx \sin \theta \approx \tan \theta = \frac{s}{L}$  where  $s$  = separation (of dots in this case)  
 $L$  = distance to observer

$$\therefore \frac{s}{L_{\text{max}}} = 1.22 \frac{\lambda}{D}$$



Given that space between dots is considerably smaller than the dot size  $\Rightarrow s \approx 2r = d$   
 dot diameter

$$\therefore L_{\text{max}} = \frac{dD}{1.22\lambda}$$

For eye pupil,  $D = 4.00 \text{ mm} \Rightarrow L_{\text{max}} = \frac{(0.00250 \text{ m})(0.00400 \text{ m})}{1.22 (550 \times 10^{-9} \text{ m})} = 14.9 \text{ m}$  18.2 m  
↑

for camera,  $D = 25.0 \text{ mm} \Rightarrow L_{\text{max}} = \frac{(0.00250 \text{ m})(0.0250 \text{ m})}{1.22 (550 \times 10^{-9} \text{ m})} = 93.1 \text{ m}$  119.6 m  
↑  
with the  
1.22 removed  
as in Eqn 38-14  
- also  
accepted  
answers

4. Consider the following observations of three observers sitting on a train platform watching a fast-moving train pass by. All three observers have properly synchronized (已同步) their watches.

The train is traveling at  $v = 0.600 c$ . ( $1 \text{ nsec} = 1 \text{ nanosecond} = 10^{-9} \text{ sec}$ )

- Observer A:** "I was sitting on a platform bench marked  $x = 61.0 \text{ m}$  when through the window of the train right in front of me I saw **Evil Student** push a button on a box marked **Destruction Ray**. My watch read  $t = 100.0 \text{ nsec}$ ."
- Observer B:** "I was sitting on a platform bench marked  $x = 76.0 \text{ m}$  when through the window of the train right in front of me I saw all the homework solution sets **Poor Student** was holding burst into flame. My watch read  $t = 230.0 \text{ nsec}$ ."
- Observer C:** "I was sitting on a platform bench marked  $x = 30.5 \text{ m}$  when through a window of the train right in front of me I saw all of **LAO BI's** clothes burn up, leaving him naked. My watch read  $t = 160.0 \text{ nsec}$ ."

- (a) (5 points) What are the coordinates of **Evil Student** pushing the button in the rest frame of the train ( $x', t'$ )? Assume  $(x', t') = (0, 0)$  when  $(x, t) = (0, 0)$ .
- (b) (5 points) **Evil Student** has been charged with destroying the solution sets of **Poor Student** and with offending the train passengers by making **LAO BI's** clothes burn up. Can you **show** that **Evil Student** must be innocent (天真无罪) of either of these two charges?

(a) Lorentz transformation:  $x' = \gamma(x - vt)$   
 $t' = \gamma(t - vx/c^2)$   
 $v = 0.6c \Rightarrow \gamma = \frac{1}{\sqrt{1 - (0.6)^2}} = \frac{1}{\sqrt{1 - 0.36}} = \frac{1}{\sqrt{0.64}} = \frac{1}{0.8} = \frac{5}{4}$

$x_A = 61.0 \text{ m}$   
 $t_A = 100.0 \text{ ns}$   
 $x'_A = \frac{5}{4} [61.0 - 0.6(100 \times 10^{-9})(3 \times 10^8)] = 53.75 \text{ m}$   
 $t'_A = \frac{5}{4} [100 \times 10^{-9} - \frac{0.6(61.0)}{3 \times 10^8}] = -27.5 \text{ ns}$

(not required) but just FYI:  $x_B = 76 \text{ m}$   
 $t_B = 230 \text{ ns}$   
 $x'_B = 43.25 \text{ m}$   
 $t'_B = 97.5 \text{ ns}$   
 $x_C = 30.5 \text{ m}$   
 $t_C = 160 \text{ ns}$   
 $x'_C = 2.8 \text{ m}$   
 $t'_C = 123.75 \text{ ns}$

(b) one test is to check speed of required signal:

$v_{BA} = \frac{x_B - x_A}{t_B - t_A} = \frac{76 \text{ m} - 61 \text{ m}}{230 \text{ ns} - 100 \text{ ns}} = \frac{15 \text{ m}}{130 \text{ ns}} \approx 1.15 \times 10^8 \text{ m/s} \approx 0.385 c$

$v_{CA} = \frac{x_C - x_A}{t_C - t_A} = \frac{30.5 \text{ m} - 61 \text{ m}}{160 \text{ ns} - 100 \text{ ns}} = \frac{-30.5 \text{ m}}{60 \text{ ns}} \approx -5.083 \times 10^8 \text{ m/s} \approx -1.694 c$

$|v_{CA}| > c \Rightarrow$  impossible according to Mr. Einstein

Equivalently, you can check the Lorentz invariant

$\Delta S_{BA}^2 = -c^2(t_B - t_A)^2 + (x_B - x_A)^2 = -12.9 \text{ m}^2 < 0 \Rightarrow$  time-like separated

$\Delta S_{CA}^2 = -c^2(t_C - t_A)^2 + (x_C - x_A)^2 = 606.25 \text{ m}^2 > 0 \Rightarrow$  space-like separated

$\Uparrow$   
 will get same  $t$ 's in train rest frame

$\Rightarrow$  A cannot cause C

$\therefore$  Evil student pushing button cannot be responsible for  $t$ 's clothes disappearing

(Problem 4 continued on next page)

Notes: if you did these calculations in the train's reference (rest)-frame you would get  
 $v'_{BA} = -0.385 c$   
 $v'_{CA} = -1.694 c$   
 $\Downarrow$   
 same conclusion

(Problem #4)  
(continued)

- (c) (4 points) Find a train speed for which the **Destruction Ray** button is pushed at the same time in the train's rest frame as one of the other events.  
 (d) (4 points) Find a train speed for which the **Destruction Ray** button is pushed at the same place in the train's rest frame as one of the other events.

(c) A cannot cause C  $\Leftrightarrow$  space-like separated  $\Rightarrow$  A and C can happen at the same time in different places

if train speed is  $v''$

$$\left. \begin{aligned} t_2'' &= \gamma'' \left( t_2 - \frac{v'' x_2}{c^2} \right) \\ t_A'' &= \gamma'' \left( t_A - \frac{v'' x_A}{c^2} \right) \end{aligned} \right\} \Delta t_{CA}'' = t_2'' - t_A'' = 0$$

$$\Leftrightarrow \gamma'' \left( t_2 - t_A - \frac{v'' (x_2 - x_A)}{c^2} \right) = 0$$

$\gamma'' \neq 0$

$$\Rightarrow t_2 - t_A = \frac{v'' (x_2 - x_A)}{c^2} \Rightarrow v'' = \frac{t_2 - t_A}{x_2 - x_A} c^2 = \frac{60 \text{ ns}}{-30.5 \text{ m}} (3 \times 10^8 \text{ m/s})^2 = 0.590164c \approx 1.77 \times 10^8 \text{ m/s}$$

$$\underline{\underline{v'' \approx 0.590c \approx 1.77 \times 10^8 \text{ m/s}}}$$

(d) Event B and Event A can happen in the same place but at different times if train speed is  $v'''$

$$\left. \begin{aligned} x_B''' &= \gamma''' (x_B - v''' t_B) \\ x_A''' &= \gamma''' (x_A - v''' t_A) \end{aligned} \right\} \Delta x_{BA}''' = x_B''' - x_A''' = 0 \Rightarrow \gamma''' (x_B - x_A - v''' (t_B - t_A)) = 0$$

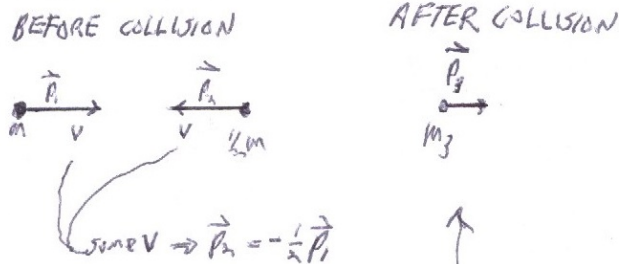
$\gamma''' \neq 0$

$$\Rightarrow x_B - x_A = v''' (t_B - t_A) \Rightarrow v''' = \frac{x_B - x_A}{t_B - t_A} = \frac{15 \text{ m}}{130 \text{ ns}} = 1.15385 \times 10^8 \text{ m/s} \approx 0.384615c$$

$$\underline{\underline{v''' \approx 0.385c \approx 1.15 \times 10^8 \text{ m/s}}}$$

[note: this is  $v_{BA}$   
at part (b)]

5. (14 points) A particle of (rest) mass  $m$  moving along the  $x$ -axis with velocity  $v$  collides with a particle of (rest) mass  $\frac{1}{2}m$  moving along the  $x$ -axis with a velocity  $-v$ . This results in the formation of one new particle. What is the rest mass of this newly-formed particle? Check that this gives the expected result for  $v \ll c$ .



momentum conservation:  $\vec{p}_1 + \vec{p}_2 = \vec{p}_3 \Rightarrow \vec{p}_1 - \frac{1}{2}\vec{p}_1 = \vec{p}_3 \Rightarrow \vec{p}_3 = +\frac{1}{2}\vec{p}_1$

(relativistic)

energy conservation:  $E_1 + E_2 = E_3 \Rightarrow \sqrt{c^2 p_1^2 + m^2 c^4} + \sqrt{c^2 p_2^2 + (\frac{1}{2}m)^2 c^4} = \sqrt{c^2 p_3^2 + m_3^2 c^4}$

or

$p_2 = -\frac{1}{2}p_1$  and  $m_3 = \frac{1}{2}m$  solve for:  $\sqrt{c^2 p_1^2 + m^2 c^4} + \sqrt{c^2 (\frac{1}{2}p_1)^2 + (\frac{1}{2}m)^2 c^4} = \sqrt{c^2 (\frac{1}{2}p_1)^2 + m_3^2 c^4}$

$\downarrow$   
 $p_1 = \gamma m v$   
 $\Rightarrow \frac{3}{2} \sqrt{c^2 p_1^2 + m^2 c^4} = \sqrt{\frac{1}{4} c^2 p_1^2 + m_3^2 c^4}$

$\Rightarrow \frac{9}{4} (c^2 p_1^2 + m^2 c^4) = \frac{1}{4} c^2 p_1^2 + m_3^2 c^4 \Rightarrow m_3^2 c^4 = 2 c^2 p_1^2 + \frac{9}{4} m^2 c^4$   
 $\quad \quad \quad = 2 c^2 \gamma^2 m^2 v^2 + \frac{9}{4} m^2 c^4$   
 $\quad \quad \quad = m^2 c^4 \left[ \frac{2 v^2}{c^2 (1 - \frac{v^2}{c^2})} + \frac{9}{4} \right]$

alternative derivation:

$\vec{p}_3 = \frac{1}{2} \vec{p}_1 = \frac{1}{2} \gamma m \vec{v}$

$E_3 = E_1 + E_2 = \gamma m c^2 + \gamma (\frac{1}{2}m) c^2 = \frac{3}{2} \gamma m c^2$



$m_3^2 c^4 = E_3^2 - p_3^2 c^2 = \frac{9}{4} \gamma^2 m^2 c^4 - \frac{1}{4} \gamma^2 m^2 v^2$

$m_3^2 c^4 = \frac{m^2 c^4}{4} \frac{9 c^2 - v^2}{c^2 - v^2}$

$\downarrow$   
 $m_3 = \frac{m}{2} \sqrt{\frac{9 - v^2/c^2}{1 - v^2/c^2}}$

for  $v \ll c$ ,

$m_3 = \frac{m}{2} \sqrt{\frac{9}{1}} = \frac{3}{2} m = m + \frac{1}{2} m$  as expected if mass is conserved