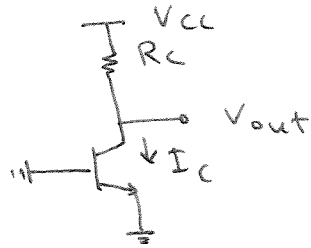
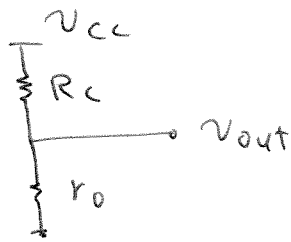


(1)

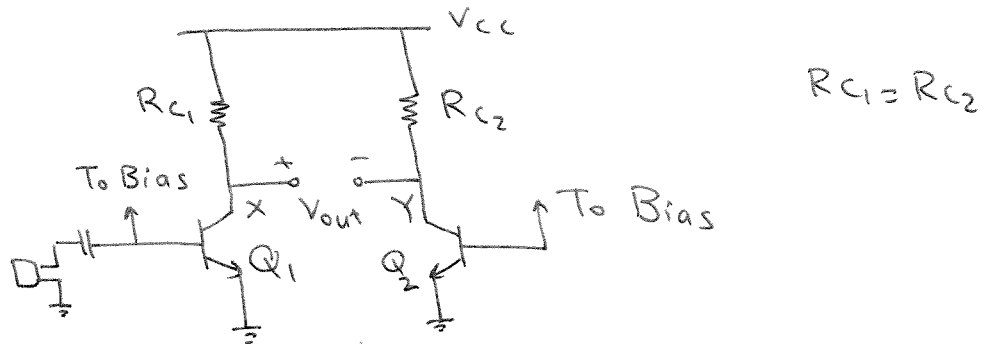


the small signal model is as follow:

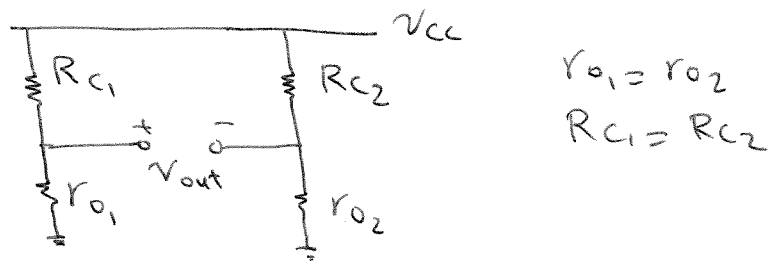


$$\frac{v_{out}}{v_{cc}} = \frac{r_o}{r_o + R_c} = \frac{\frac{V_A}{I_c}}{\frac{V_A}{I_c} + R_c} = \frac{V_A}{V_A + R_c I_c}$$

(2)



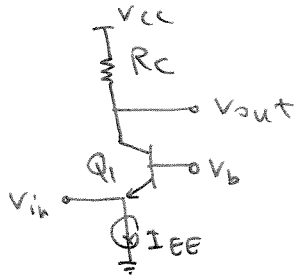
The small signal model is:



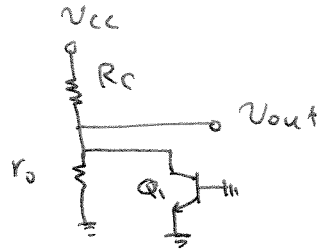
$$\frac{V_{out}}{V_{CC}} = \frac{1}{V_{CC}} \left(\frac{r_{o1}}{R_{C1} + r_{o1}} - \frac{r_{o2}}{R_{C2} + r_{o2}} \right) V_{CC} = 0$$

(3)

(a)

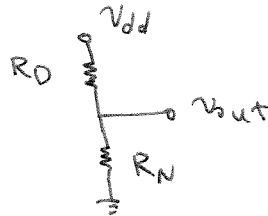
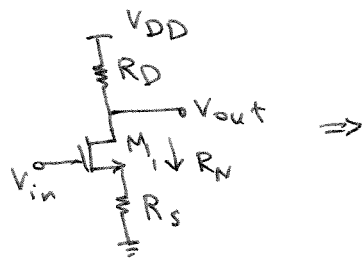


The small signal model is:



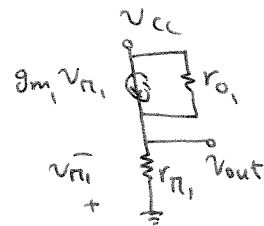
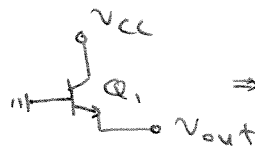
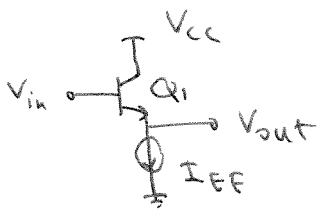
$$\frac{v_{out}}{v_{cc}} = \frac{r_o}{r_o + R_C} = \frac{V_A}{V_A + R_C I_{EE}}$$

(b)



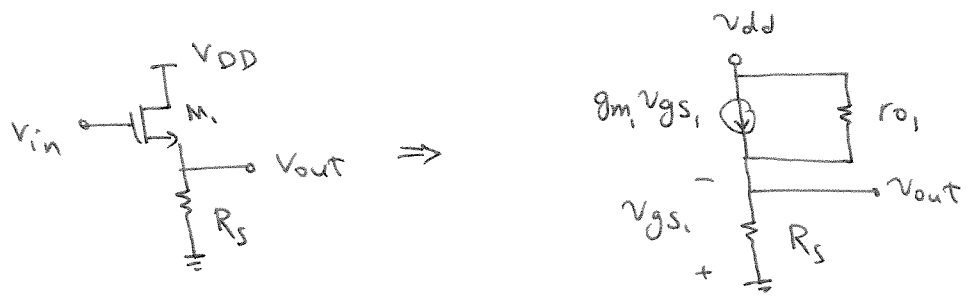
$$R_N = g_{m1} r_{o1} R_S + r_{o1} + R_S, \quad \frac{v_{out}}{v_{dd}} = \frac{R_N}{R_N + R_D}$$

(c)



$$\frac{v_{out}}{r_{\pi 1}} + \frac{v_{out} - v_{cc}}{r_{o1}} + g_{m1} v_{out} = 0 \Rightarrow (r_{\pi 1} + r_{o1} + g_{m1} r_{o1} r_{\pi 1}) v_{out} = \frac{r_{\pi 1}}{\beta r_{o1} + r_{o1} + r_{\pi 1}} v_{cc}$$

(3) (d)

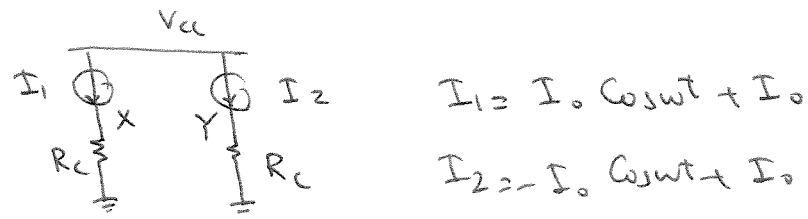


$$\frac{V_{out}}{R_S} + \frac{V_{out} - V_{DD}}{r_{o1}} + g_m V_{out} = 0$$

$$\Rightarrow (r_{o1} + R_S + g_m r_{o1} R_S) V_{out} = R_S V_{DD}$$

$$\Rightarrow \frac{V_{out}}{V_{DD}} = \frac{R_S}{g_m r_{o1} R_S + R_S + r_{o1}}$$

④

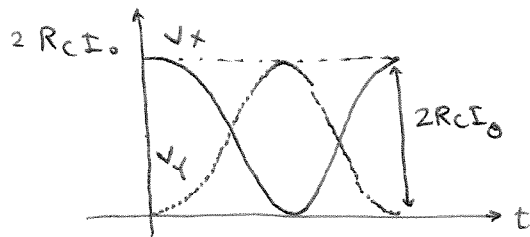


$$I_1 = I_0 \cos \omega t + I_0$$

$$I_2 = -I_0 \cos \omega t + I_0$$

$$V_X = R_C I_1 = R_C I_0 (1 + \cos \omega t)$$

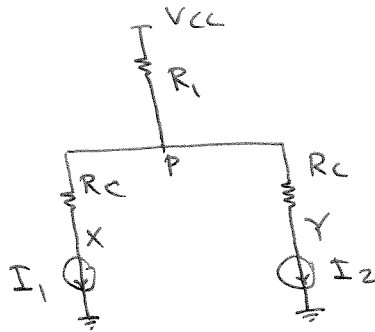
$$V_Y = R_C I_2 = R_C I_0 (1 - \cos \omega t)$$



$$V_{X,P-P} = V_{Y,P-P} = 2R_C I_0$$

$$V_{X,CM} = V_{Y,CM} = R_C I_0$$

⑤



$$I_1 = I_0 \cos \omega t + I_0$$

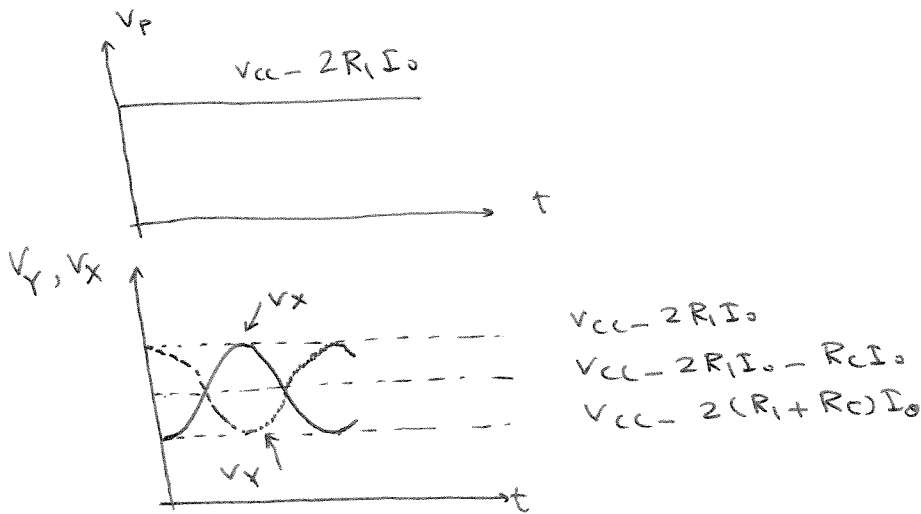
$$I_2 = -I_0 \cos \omega t + I_0$$

$$V_P = V_{CC} - R_1 (I_1 + I_2) = V_{CC} - 2 R_1 I_0$$

$$V_X = V_P - R_C I_1 = V_{CC} - 2 R_1 I_0 - R_C I_0 - R_C I_0 \cos \omega t$$

$$\Rightarrow V_X = V_{CC} - (2 R_1 + R_C) I_0 - R_C I_0 \cos \omega t$$

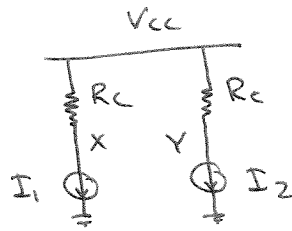
$$V_Y = V_P - R_C I_2 = V_{CC} - (2 R_1 + R_C) I_0 + R_C I_0 \cos \omega t$$



$$V_{X,CM} = V_{Y,CM} = V_{CC} - (2 R_1 + R_C) I_0$$

$$V_{X,P-P} = V_{Y,P-P} = 2 R_C I_0$$

⑥

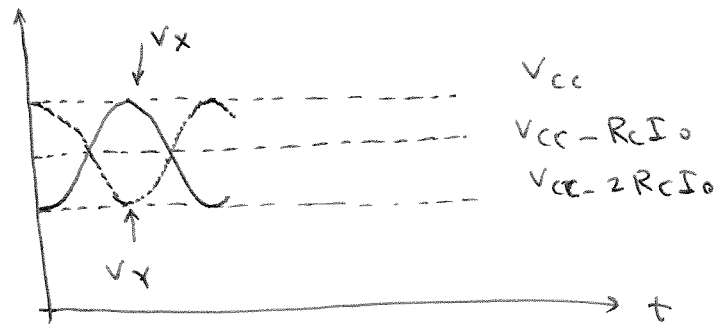


$$I_1 = I_0 \cos \omega t + I_0$$

$$I_2 = -I_0 \cos \omega t + I_0$$

$$V_X = V_{CC} - R_C I_1 = V_{CC} - R_C I_0 (1 + \cos \omega t)$$

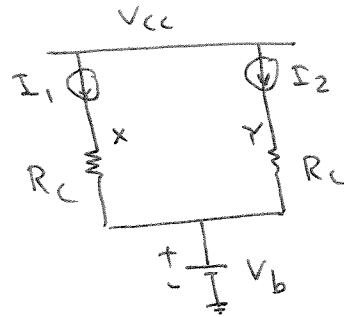
$$V_Y = V_{CC} - R_C I_2 = V_{CC} - R_C I_0 (1 - \cos \omega t)$$



$$V_{X,CM} = V_{Y,CM} = V_{CC} - R_C I_0$$

$$V_{X,P-P} = V_{Y,P-P} = 2 R_C I_0$$

⑦

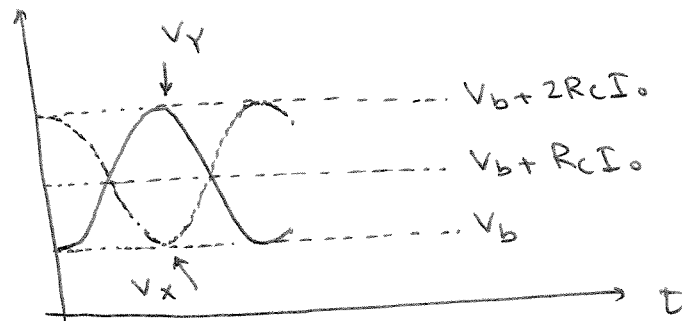


$$I_1 = I_0 \cos \omega t + I_0$$

$$I_2 = -I_0 \cos \omega t + I_0$$

$$V_X = R_C I_1 + V_b = R_C I_0 (1 + \cos \omega t) + V_b$$

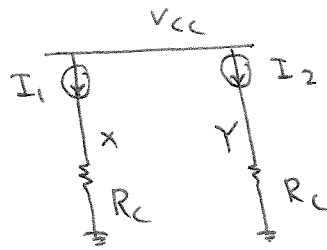
$$V_Y = R_C I_2 + V_b = R_C I_0 (1 - \cos \omega t) + V_b$$



$$V_{X,CM} = V_{Y,CM} = V_b + R_C I_0$$

$$V_{X,P-P} = V_{Y,P-P} = 2R_C I_0$$

(8)

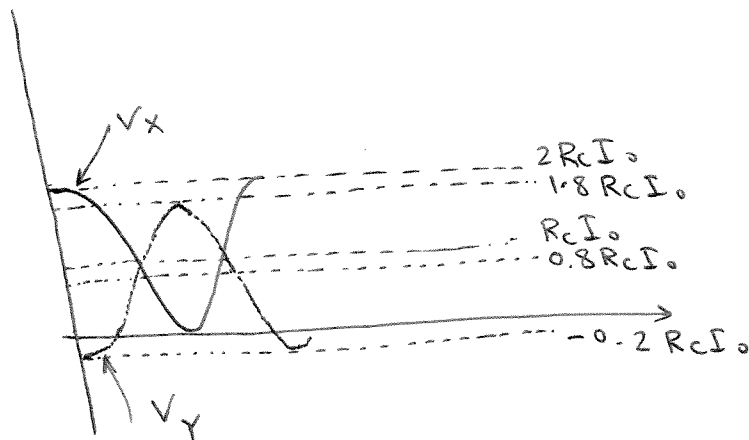


$$I_1 = I_0 \cos \omega t + I_0$$

$$I_2 = -I_0 \cos \omega t + 0.8 I_0$$

$$V_X = R_C I_1 = R_C I_0 (1 + \cos \omega t)$$

$$V_Y = R_C I_2 = R_C I_0 (0.8 - \cos \omega t)$$



$$V_{X,CM} = R_C I_0 \quad V_{Y,CM} = 0.8 R_C I_0$$

$$V_{X,P-P} = 2 R_C I_0 \quad V_{Y,P-P} = 2 R_C I_0$$

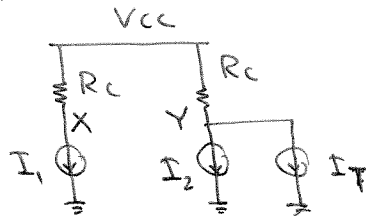
X and Y are true differential signals.

9)

$$I_1 = I_0 \cos \omega t + I_0$$

$$I_2 = -I_0 \cos \omega t + I_0$$

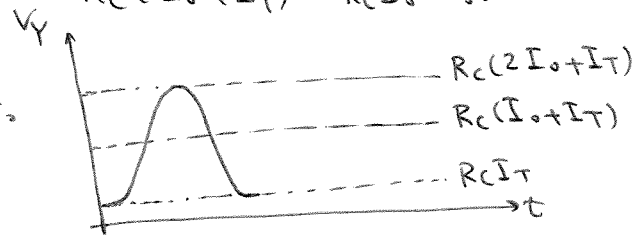
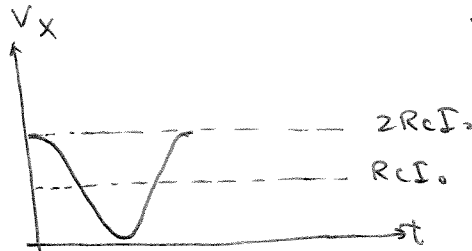
(a)



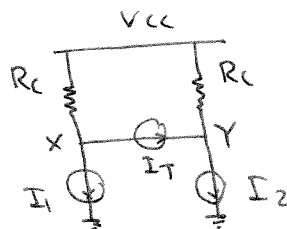
$$V_x = R_c I_1 = R_c I_0 (1 + \cos \omega t)$$

$$V_y = R_c (I_2 + I_T) =$$

$$R_c (I_0 + I_T) - R_c I_0 \cos \omega t$$

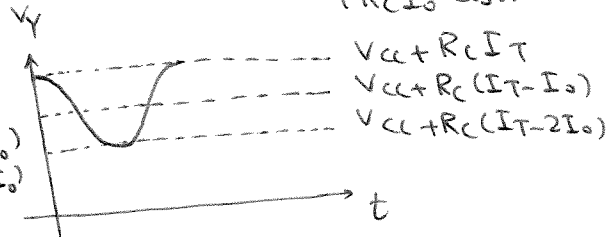
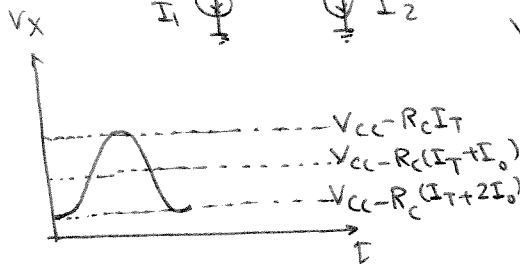


(b)

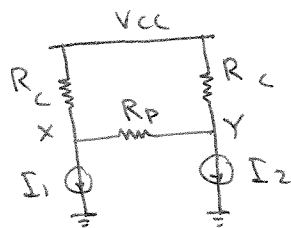


$$V_x = V_{cc} - R_c (I_1 + I_T) = V_{cc} - R_c (I_0 + I_T) - R_c I_0 \cos \omega t$$

$$V_y = V_{cc} - R_c (I_2 - I_T) = V_{cc} - R_c (I_0 - I_T) + R_c I_0 \cos \omega t$$



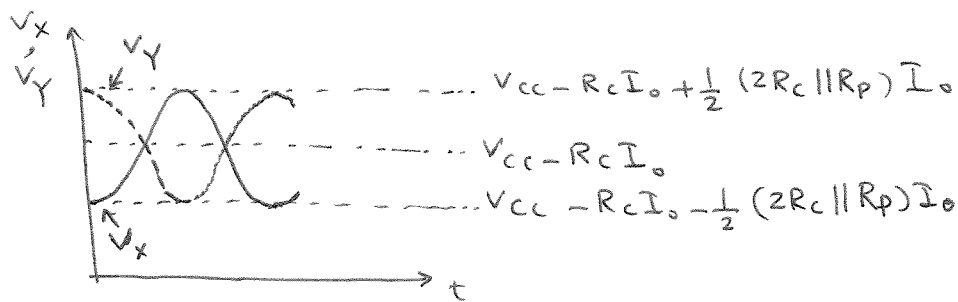
(c)



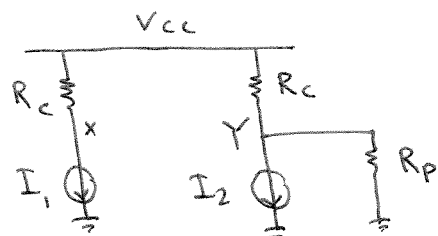
$$\begin{cases} \frac{V_x - V_{cc}}{R_c} + \frac{V_x - V_y}{R_p} + I_1 = 0 \\ \frac{V_y - V_{cc}}{R_c} + \frac{V_y - V_x}{R_p} + I_2 = 0 \end{cases} \Rightarrow$$

$$V_x = V_{cc} - R_c I_0 - \frac{R_c R_p}{2R_c + R_p} I_0 \cos \omega t$$

$$V_y = V_{cc} - R_c I_0 + \frac{R_c R_p}{2R_c + R_p} I_0 \cos \omega t$$



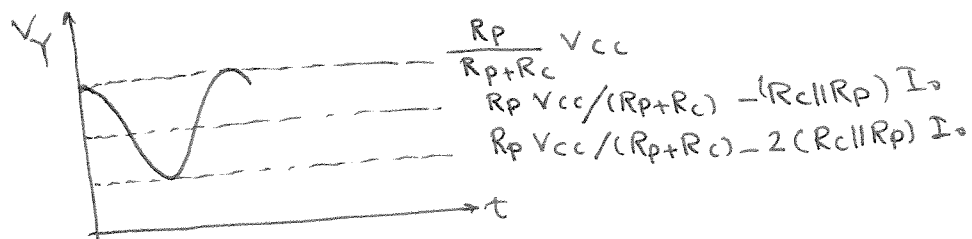
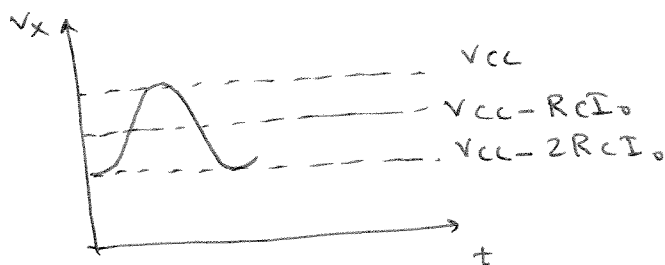
(d)



$$V_x = V_{CC} - R_c I_1 = V_{CC} - R_c I_o (1 + \cos \omega t)$$

$$V_Y = \frac{R_p}{R_p + R_c} V_{CC} - (R_c \parallel R_p) I_2 =$$

$$\frac{R_p}{R_p + R_c} V_{CC} - (R_c \parallel R_p) I_o (1 - \cos \omega t)$$

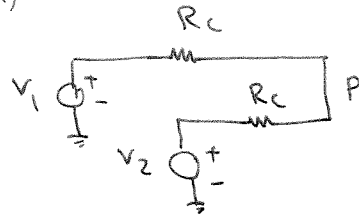


40)

$$V_1 = V_0 \cos \omega t + V_0$$

$$V_2 = -V_0 \cos \omega t + V_0$$

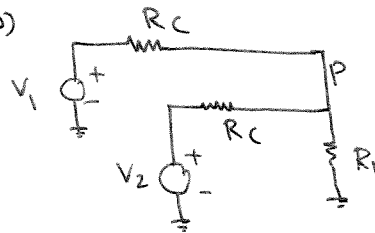
(a)



$$V_p = \frac{V_1 + V_2}{2} = V_0$$



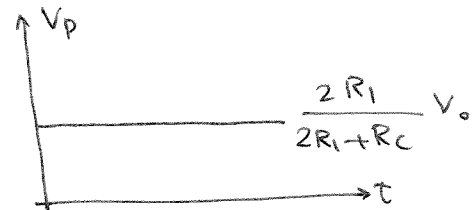
(b)



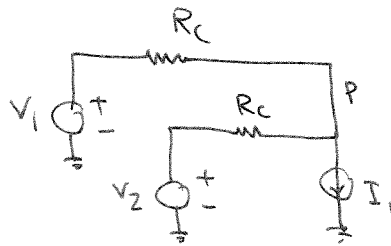
$$\frac{2V_p - V_1 - V_2}{R_c} + \frac{V_p}{R_i} = 0 \Rightarrow$$

$$\frac{2V_p - 2V_0}{R_c} + \frac{V_p}{R_i} = 0 \Rightarrow (2R_i + R_c)V_p = 2V_0 R_i$$

$$\rightarrow V_p = \frac{2R_i}{2R_i + R_c} V_0$$



(c)



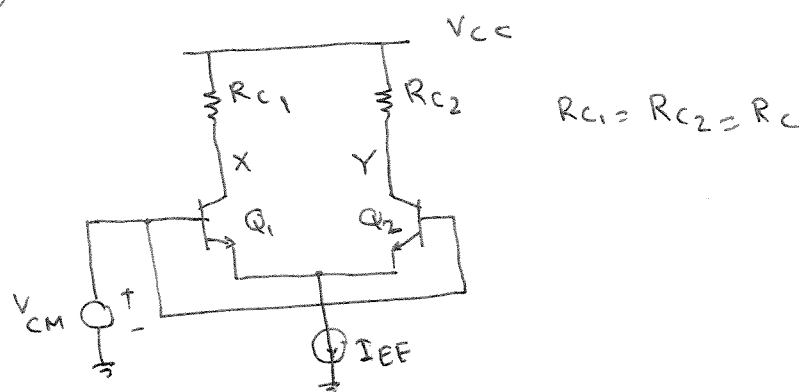
$$\frac{2V_p - V_1 - V_2}{R_c} + I_1 = 0 \Rightarrow$$

$$\frac{2V_p - 2V_0}{R_c} = -I_1 \Rightarrow$$

$$V_p = V_0 - \frac{R_c I_1}{2}$$



(11)



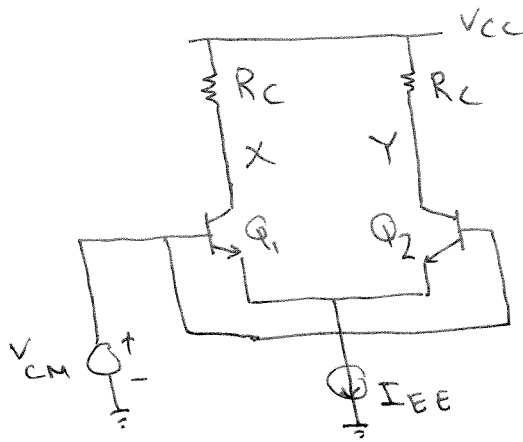
if V_{CC} changes by ΔV then:

$$\Delta V_X = \Delta V, \quad \Delta V_Y = \Delta V \Rightarrow$$

$$\Delta (V_X - V_Y) = 0$$

Since both V_X and V_Y response to the supply changes similarly, the circuit rejects supply noise.

12

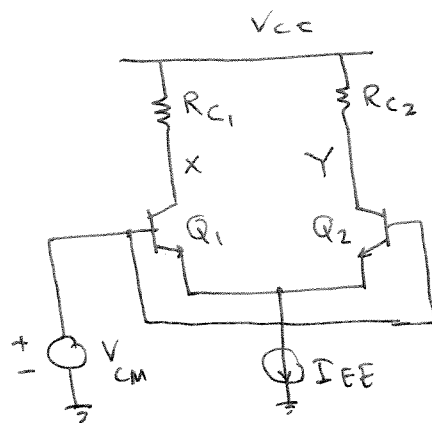


$$\Delta V_X = - \frac{R_C \Delta I_{EE}}{2} = - \frac{R_C \Delta I}{2}$$

$$\Delta V_Y = - \frac{R_C \Delta I_{EE}}{2} = - \frac{R_C \Delta I}{2}$$

$$\Rightarrow \Delta(V_X - V_Y) = 0$$

(13)



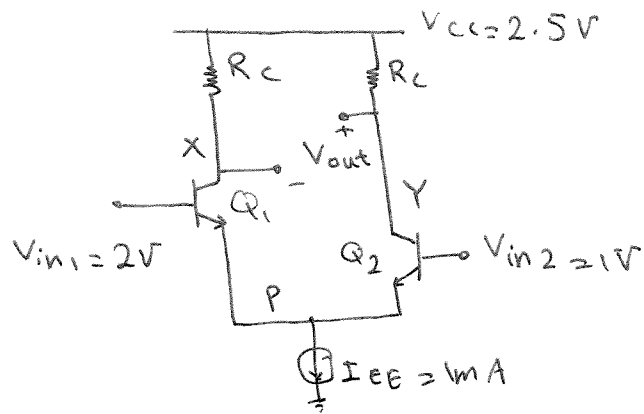
$$R_{C1} = R_{C2} + \Delta R$$

$$\Delta V_X = - \frac{R_{C1} \Delta I}{2} = - \frac{(R_{C2} + \Delta R) \Delta I}{2}$$

$$\Delta V_Y = - \frac{R_{C2} \Delta I}{2} \Rightarrow$$

$$\Delta(V_X - V_Y) = - \frac{\Delta R \Delta I}{2}$$

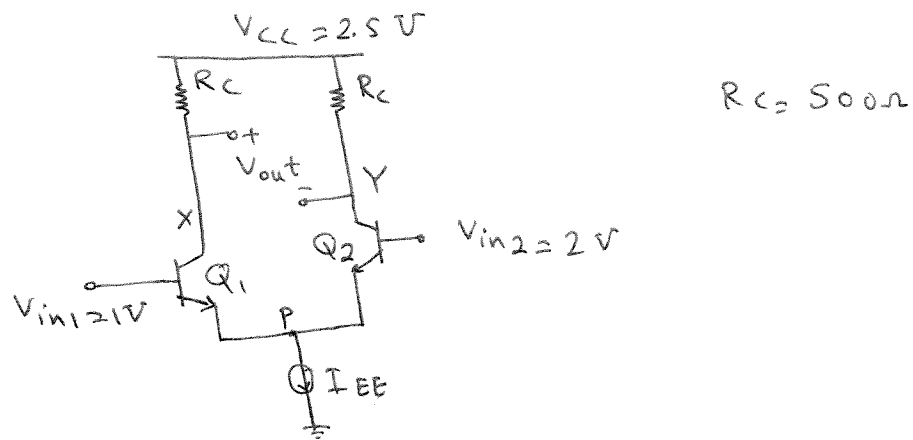
14



$$V_X \geq V_{in1} \Rightarrow V_X \geq 2 \Rightarrow V_{CC} - R_C I_{EE} \geq 2$$

$$\Rightarrow 2.5 - R_C^{(k\Omega)} \geq 2 \Rightarrow R_C \leq 0.5\text{ k}\Omega$$

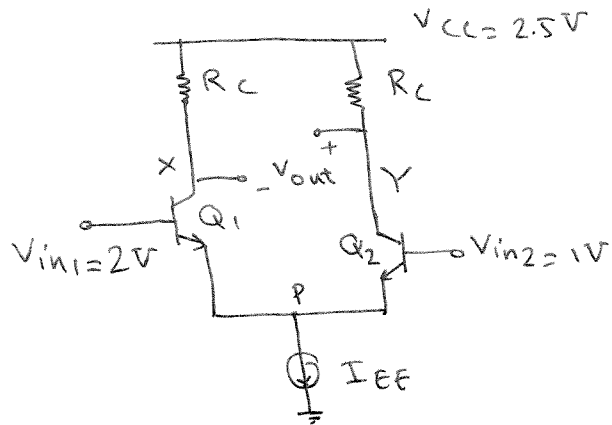
15



$$V_Y \geq V_{in2} \Rightarrow V_{CC} - R_C I_{EE} \geq 2 \Rightarrow$$

$$2.5 - 500 I_{EE} \geq 2 \Rightarrow I_{EE} \leq 1\text{ mA}$$

16



$$I_{EE} = 1mA$$

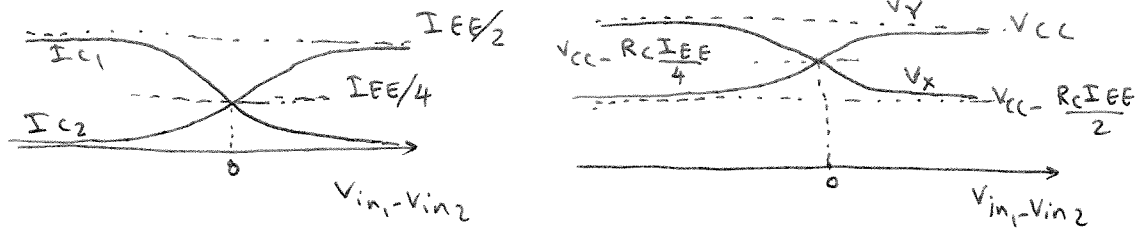
$$R_C = 800\Omega$$

$$V_X = V_{CC} - R_C I_{EE} = 2.5 - 0.8 = 1.7V$$

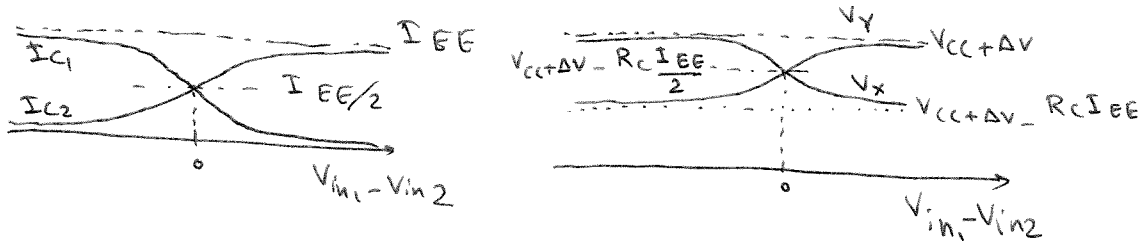
$\Rightarrow V_X < V_{in1} \Rightarrow Q_1$ is in saturation region.

(17)

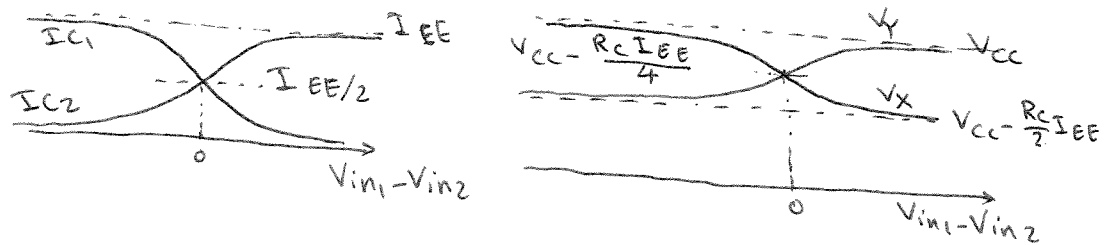
(a)



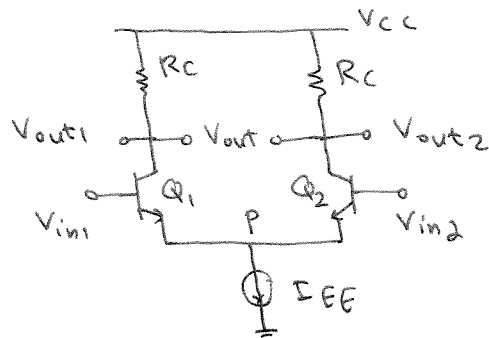
(b)



(c)



18



$$\frac{I_{C1}}{I_{C2}} = 5$$

$$V_{in1} - V_{in2} = V_T \ln \frac{I_{C1}}{I_{C2}} = 0.026 \ln 5 = 41.845 \text{ mV}$$

at 27° , $V_T = 26 \text{ mV} \Rightarrow$ at 100° ,

$$V_T = \frac{(273 + 100)}{273 + 27} 26^{\text{mV}} = 32.33 \text{ mV}$$

$$\Rightarrow \frac{41.845}{\text{mV}} = \frac{32.33}{\text{mV}} \ln \frac{I_{C1}}{I_{C2}} \Rightarrow \frac{I_{C1}}{I_{C2}} = 3.65$$

(19)

$$I_{C2} = I_{C1} = \frac{I_{EE}}{2}$$

if I_{C2} changes by 10% then

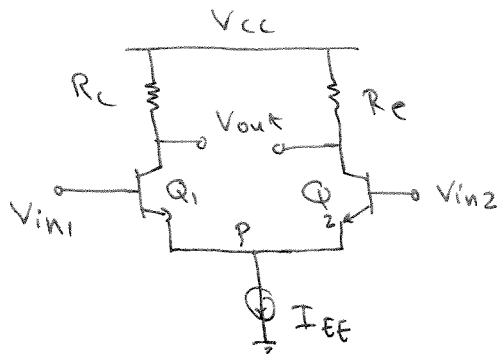
$$1.1 \times I_{C2 \text{ bias}} = \frac{I_{EE}}{1 + \exp \frac{V_{in1} - V_{in2}}{V_T}} \Rightarrow$$

$$1.1 \times \frac{I_{EE}}{2} = \frac{I_{EE}}{1 + \exp \frac{V_{in1} - V_{in2}}{V_T}} \Rightarrow$$

$$V_{in1} - V_{in2} = V_T \ln \frac{0.9}{1.1} = -0.2 V_T = -5.217 \text{ mV}$$

So the input differential voltage should change by no more than 5.2 mV.

(20)



$$I_{C2} = \frac{I_{EE}}{1 + \exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)}$$

$$I_{C2 \text{ bias}} = \frac{I_{EE}}{2}$$

if the transconductance of Q_2 drops by a factor of 2, then $I_{C2} = \frac{I_{EE}}{4}$

$$\Rightarrow \frac{I_{EE}}{4} = \frac{I_{EE}}{1 + \exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)} \Rightarrow$$

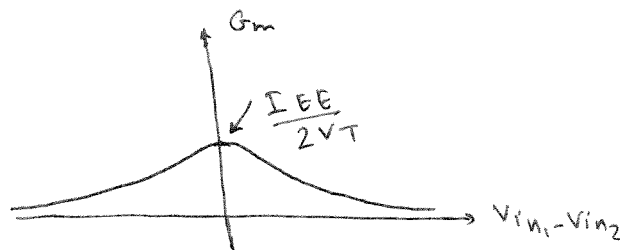
$$V_{in1} - V_{in2} = V_T \ln 3 = 1.0986 V_T = 28.564 \text{ mV}$$

$$(21) \quad v_{in1} - v_{in2} = \Delta v_{in}$$

$$I_{C1} - I_{C2} = \frac{I_{EE} \exp \frac{\Delta v_{in}}{V_T}}{1 + \exp \frac{\Delta v_{in}}{V_T}} - \frac{I_{EE}}{1 + \exp \frac{\Delta v_{in}}{V_T}}$$

$$\Rightarrow \frac{\partial (I_{C1} - I_{C2})}{\partial (\Delta v_{in})} = I_{EE} \left[\frac{\frac{1}{V_T} \exp \left(\frac{\Delta v_{in}}{V_T} \right) (1 + \exp \frac{\Delta v_{in}}{V_T}) - \frac{(\exp \frac{\Delta v_{in}}{V_T})^2}{V_T}}{(1 + \exp \frac{\Delta v_{in}}{V_T})^2} + \frac{\frac{1}{V_T} \exp \frac{\Delta v_{in}}{V_T}}{(1 + \exp \frac{\Delta v_{in}}{V_T})^2} \right]$$

$$= \frac{2 I_{EE}}{V_T} \frac{\exp \left(\frac{v_{in1} - v_{in2}}{V_T} \right)}{\left(1 + \exp \left(\frac{v_{in1} - v_{in2}}{V_T} \right) \right)^2}$$



$$\begin{cases} \max G_m = \frac{I_{EE}}{2V_T} \\ \text{At } v_{in1} - v_{in2} = 0 \end{cases}$$

$$\text{if } G_m = \frac{1}{2} G_{m_{\max}} = \frac{I_{EE}}{4V_T} \Rightarrow$$

$$\frac{\exp \left(\frac{v_{in1} - v_{in2}}{V_T} \right)}{\left(1 + \exp \left(\frac{v_{in1} - v_{in2}}{V_T} \right) \right)^2} = \frac{1}{8} \Rightarrow v_{in1} - v_{in2} = \pm 1.763 V_T = \pm 45.838 \text{ mV}$$

Q2)

$$V_{out1} - V_{out2} = -R_C I_{EE} \tanh \frac{V_{in1} - V_{in2}}{2V_T}$$

$$A_V = \frac{\partial (V_{out1} - V_{out2})}{\partial (V_{in1} - V_{in2})} = -\frac{2R_C I_{EE}}{V_T} \frac{\exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)}{\left[1 + \exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)\right]^2}$$

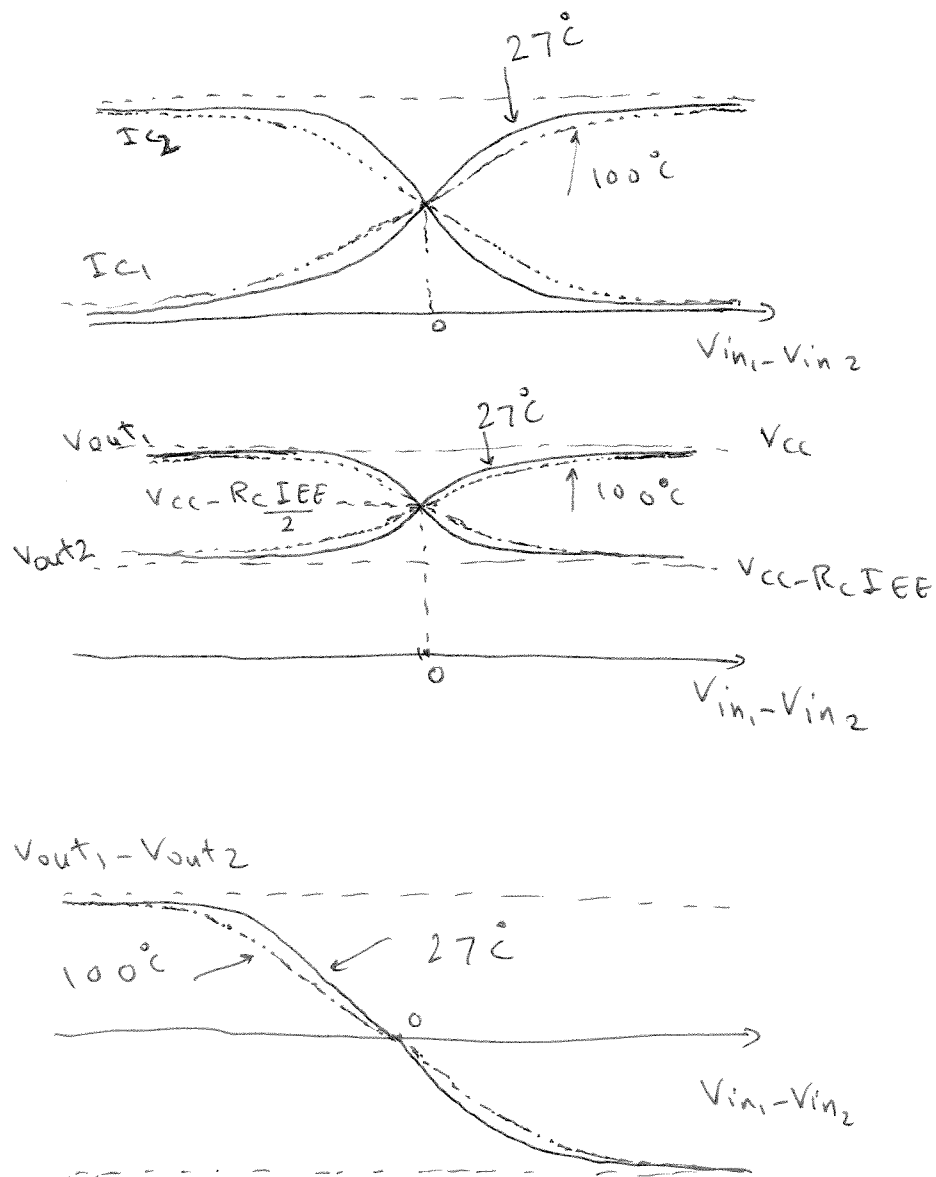
if $V_{in1} - V_{in2} = 30 \text{ mV} \Rightarrow$

$$A_V = -14.02 R_C I_{EE}$$

(23)

if the ambient temperature goes from 27°C to 100°C , V_T will change from 26mV to 32.33mV .

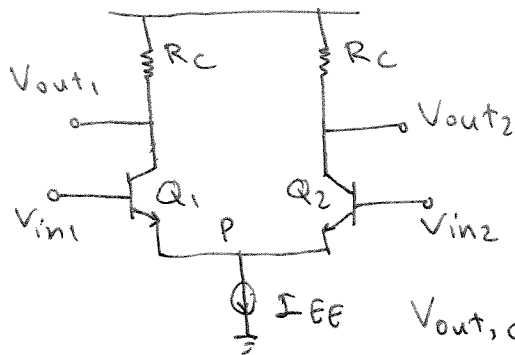
Therefore the curves stretch out as shown below to the sides:



(24) $R_C = 500 \Omega$, $I_{EE} = 1 \text{ mA}$, $V_{CC} = 2.5 \text{ V}$

$V_{in1} = V_o \sin \omega t + V_{CM}$

$V_{in2} = -V_o \sin \omega t + V_{CM}$, $V_{CM} = 1 \text{ V}$

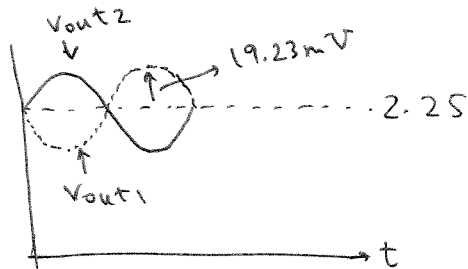


$$A_v = -g_m R_C = -\frac{I_{EE}}{2V_T} R_C =$$

$$= -\frac{10^{-3} \times 500}{2 \times 0.026} = -9.615$$

$$V_{out, CM} = V_{CC} - R_C \frac{I_{EE}}{2} = 2.5 - 0.5 \times 0.5 \Rightarrow V_{out, CM} = 2.25$$

(a) $|V_{out}| = |A_v V_{in}| = 9.615 \times 2 \text{ m} = 19.23 \text{ mV}$

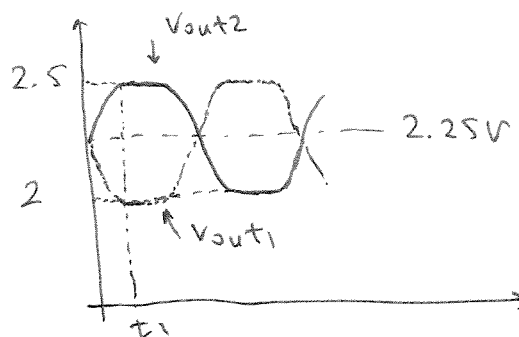


(b) $I_{C1} = 0.95 I_{EE}$, $I_{C2} = 0.05 I_{EE}$, $\frac{I_{C1}}{I_{C2}} = 19$

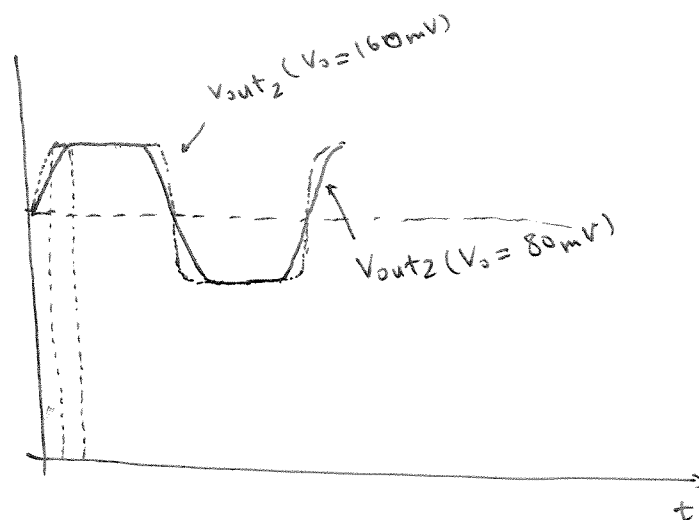
$$V_{in1} - V_{in2} = V_T \ln \frac{I_{C1}}{I_{C2}} = 76.555 \text{ mV}$$

$$\frac{V_{in1} - V_{in2}}{2} = \frac{50 \text{ mV}}{2} \sin \omega t_1 \Rightarrow 38.278 = \frac{50 \text{ mV}}{2} \sin \omega t_1 \Rightarrow$$

$$t_1 = \frac{0.872}{\omega}$$



(25)



The time at which one transistor takes 95% of the tail current source is achievable through:

$$\frac{I_{C1}}{I_{C2}} = 19 \Rightarrow V_{in1} - V_{in2} = V_T \ln \frac{I_{C1}}{I_{C2}} = 76.555 \text{ mV}$$

$$\frac{V_{in1} - V_{in2}}{2} = V_o \sin \omega t_1 \Rightarrow t_1 = \frac{\text{Arc Sin } \frac{38.278}{V_o}}{\omega}$$

evidently as V_o increases, t_1 decreases and the output waveform becomes sharper.

$$t_1(V_o = 80 \text{ mV}) = \frac{0.499}{\omega}$$

$$t_1(V_o = 160 \text{ mV}) = \frac{0.242}{\omega}$$

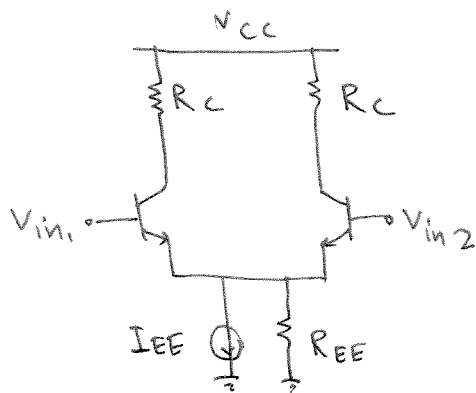
$$(26) \quad \omega = 2\pi \times (100 \text{ MHz})$$

$$\text{Slope} \approx \frac{V_{CC} - V_{CM}}{t_1} = \frac{0.25 \omega}{\text{Arc Sin}\left(\frac{38.278}{V_o \text{ (mV)}}\right)}$$

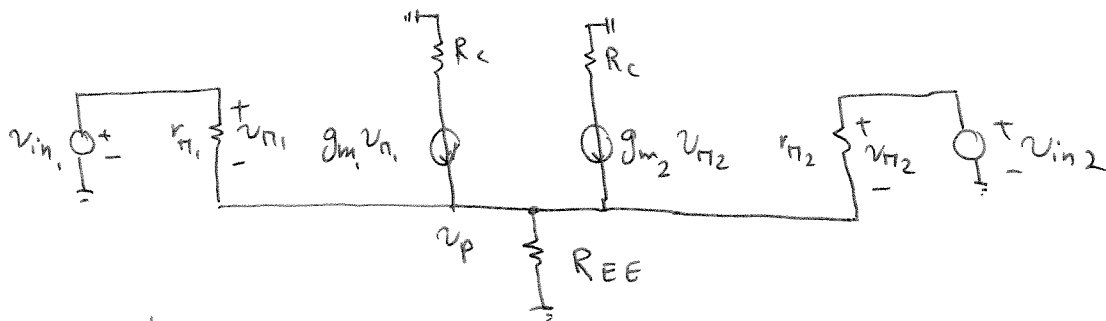
$$\Rightarrow \text{if } V_o = 80 \text{ mV} \Rightarrow \text{Slope} = 3.148 \times 10^8 \text{ V/s}$$

$$\text{if } V_o = 160 \text{ mV} \Rightarrow \text{Slope} = 6.491 \times 10^8 \text{ V/s}$$

(27)



The small signal model is,

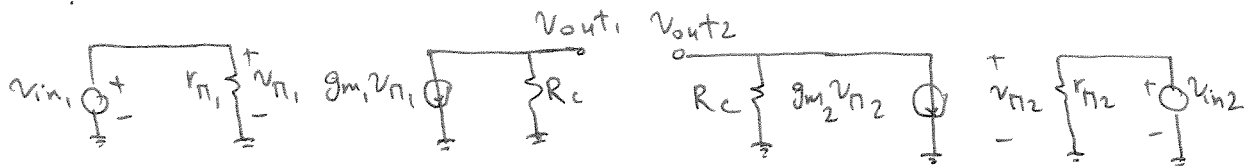


writing the node equation at P we have:

$$\frac{v_p}{R_{EE}} + \frac{v_p - v_{in1}}{r_{\pi 1}} + g_{m1}(v_p - v_{in1}) + \frac{v_p - v_{in2}}{r_{\pi 2}} + g_{m2}(v_p - v_{in2}) = 0$$

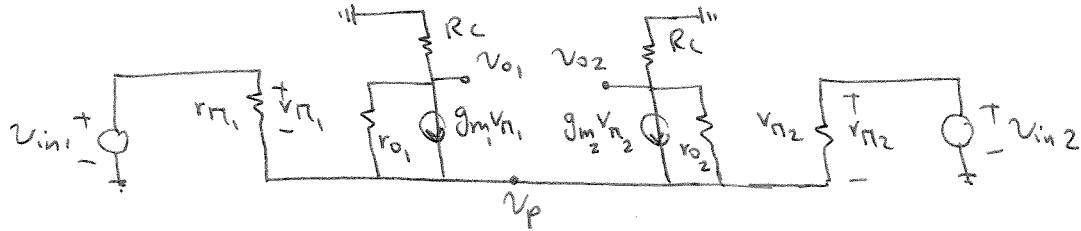
Since $v_{in1} = -v_{in2}$ and $\begin{cases} r_{\pi 1} = r_{\pi 2} \\ g_{m1} = g_{m2} \end{cases}$, the above equation simplifies to:

$$\frac{v_p}{R_{EE}} + \frac{2v_p}{r_{\pi 1}} + 2g_{m1}v_p = 0 \Rightarrow v_p = 0 \Rightarrow \text{the small signal model is:}$$



$$A_v = \frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = \frac{-g_{m1}v_{in1}R_C + g_{m2}v_{in2}R_C}{v_{in1} - v_{in2}} = -g_{m1}R_C$$

(28)



$$V_{in1} = -V_{in2} \rightarrow V_{in1} + V_{in2} = 0$$

$$g_{m1} = g_{m2}, r_{\pi1} = r_{\pi2}, r_{o1} = r_{o2}$$

Writing the node equation at V_p :

$$\frac{V_p - V_{in1}}{r_{\pi1}} + \frac{V_p - V_{o1}}{r_{o1}} + g_{m1}(V_p - V_{in1}) + g_{m2}(V_p - V_{in2}) +$$

$$\frac{V_p - V_{in2}}{r_{\pi2}} + \frac{V_p - V_{o2}}{r_{o2}} = 0 \Rightarrow 2g_{m1}V_p + \frac{2V_p - V_{o1} - V_{o2}}{r_{o1}} + \frac{2V_p \times 2}{r_{\pi1}} = 0 \quad (1)$$

Now the node equations at nodes V_{o1} and V_{o2} leads to:

$$\begin{cases} \frac{V_{o1}}{R_c} + \frac{V_{o1} - V_p}{r_{o1}} + g_{m1}(V_{in1} - V_p) = 0 \quad (2) \\ \frac{V_{o2}}{R_c} + \frac{V_{o2} - V_p}{r_{o2}} + g_{m2}(V_{in2} - V_p) = 0 \quad (3) \end{cases} \Rightarrow (2) + (3) =$$

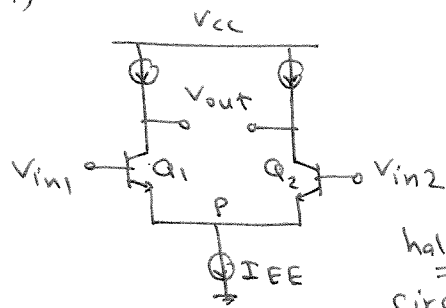
$$(V_{o1} + V_{o2})\left(\frac{1}{R_c} + \frac{1}{r_{o1}}\right) = \frac{2V_p}{r_{o1}} + 2g_{m1}V_p \quad (4)$$

placing 4 in ① $\Rightarrow$

$$2g_{m1}V_p + \frac{1}{r_{o1}}\left(2V_p - \frac{1}{\frac{1}{R_c} + \frac{1}{r_{o1}}}\left(\frac{2V_p}{r_{o1}} + 2g_{m1}V_p\right)\right) + \frac{2V_p \times 2}{r_{\pi1}} = 0$$

$$\Rightarrow \underline{V_p = 0}$$

(29)



$$I_{EE} = 1 \text{ mA}, V_A = 5 \text{ V}$$

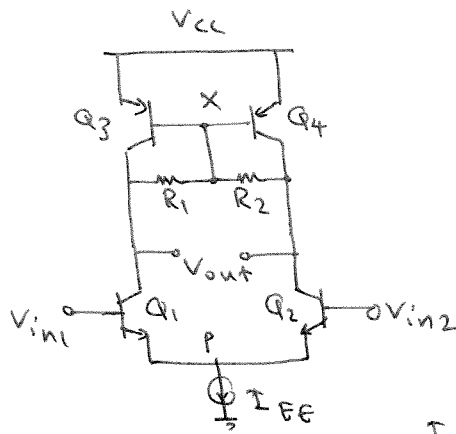
half
⇒
circuit



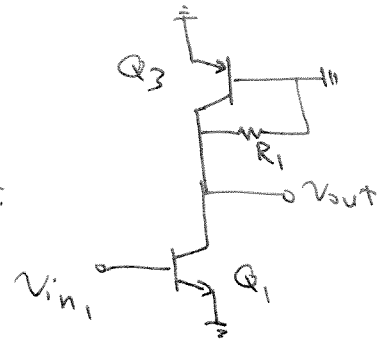
$$A_v = -g_{m1} \cdot r_{o1} = -\frac{I_{EE}}{2V_T} \cdot \frac{V_A}{\frac{I_{EE}}{2}} = -\frac{V_A}{V_T} = \frac{-5}{0.026}$$

$$\rightarrow A_v = -192.31$$

(30)



half
 $\Rightarrow$
 circuit



$$I_{EE} = 2\text{mA}, V_{A,n} = 5\text{V}, V_{A,p} = 4\text{V}$$

$$A_v = -g_{m1} (r_{o1} \parallel r_{o3} \parallel R_1)$$

$$\Rightarrow 50 = \frac{I_{EE}}{2V_T} \left(\frac{V_{A,n}}{\frac{I_{EE}}{2}} \parallel \frac{V_{A,p}}{\frac{I_{EE}}{2}} \parallel R_1 \right) \Rightarrow$$

$$50 = \frac{2}{2 \times 26} \left(\frac{5}{10^{-3}} \parallel \frac{4}{10^{-3}} \parallel R_1 \right) \rightarrow$$

$$R_1 = 3132.53\Omega$$

(31)

The half circuit is:

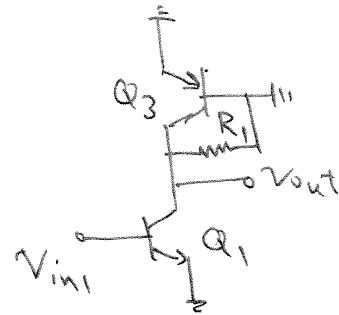
$$A_v = -g_{m1} (r_{o1} \parallel r_{o3} \parallel R_1)$$

$\Rightarrow$

$$S_o = \frac{I_{EE}}{2 \times 0.026} \left(\frac{5}{\frac{I_{EE}}{2}} \parallel \frac{4}{\frac{I_{EE}}{2}} \parallel 5K \right) \Rightarrow$$

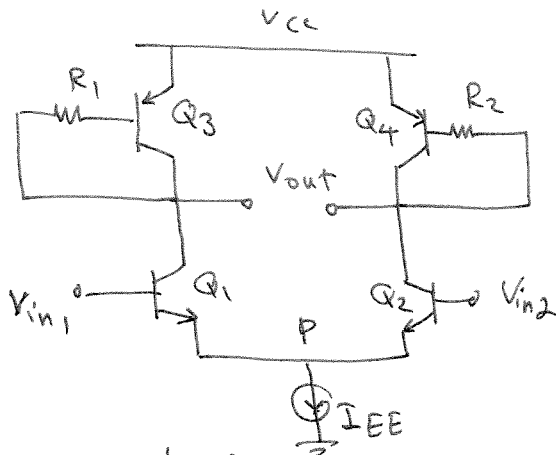
$$S_o = \frac{1}{0.052} \left(10 \parallel 8 \parallel \underset{\substack{\downarrow \\ 1K\Omega}}{5I_{EE}} \right) \Rightarrow$$

$$I_{EE} = 1.253 \text{ mA}$$

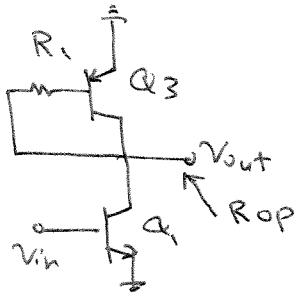


32

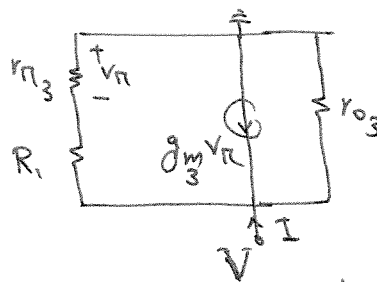
(a)



From half circuit concept we have: $A_v = -g_{m1}(r_{o1} \parallel R_{op})$



To calculate R_{op} , from small signal model we have:



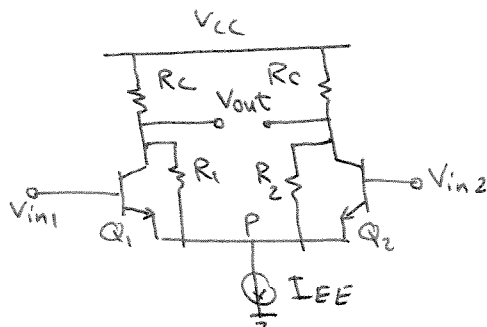
$$I = \frac{V}{r_{o3}} - g_{m3}V_{\pi} + \frac{V}{R_1 + r_{\pi 3}} = V \left[\frac{1}{r_{o3}} + \frac{1}{R_1 + r_{\pi 3}} \right] + g_{m3} \frac{r_{\pi 3}}{R_1 + r_{\pi 3}} V$$

$$\rightarrow R_{op} = \frac{V}{I} = r_{o3} \parallel (R_1 + r_{\pi 3}) \parallel \left(\left(1 + \frac{R_1}{r_{\pi 3}} \right) \frac{1}{g_{m3}} \right)$$

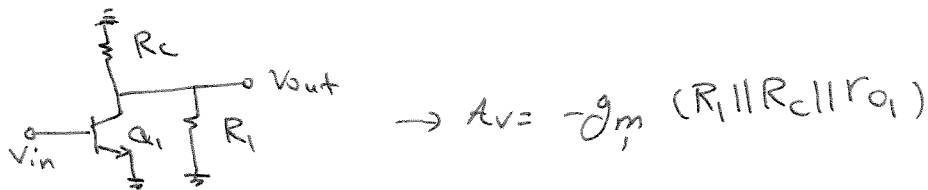
$$\rightarrow A_v = -g_{m1} \left[r_{o1} \parallel r_{o3} \parallel (R_1 + r_{\pi 3}) \parallel \left(\left(1 + \frac{R_1}{r_{\pi 3}} \right) \frac{1}{g_{m3}} \right) \right]$$

32

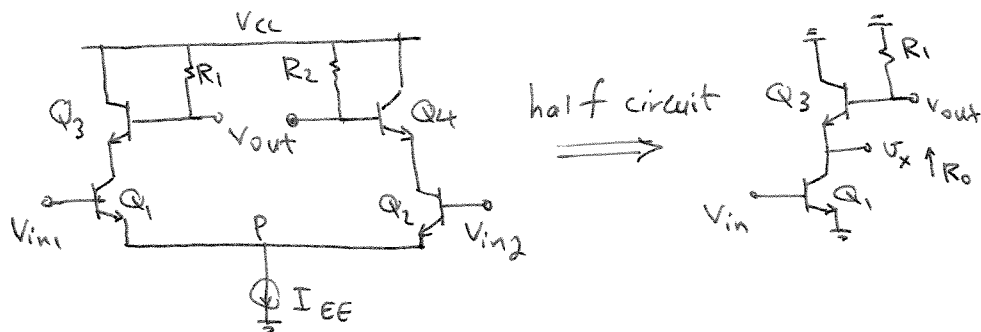
(b)



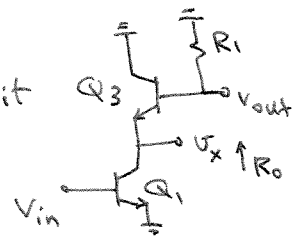
From half circuit concept:



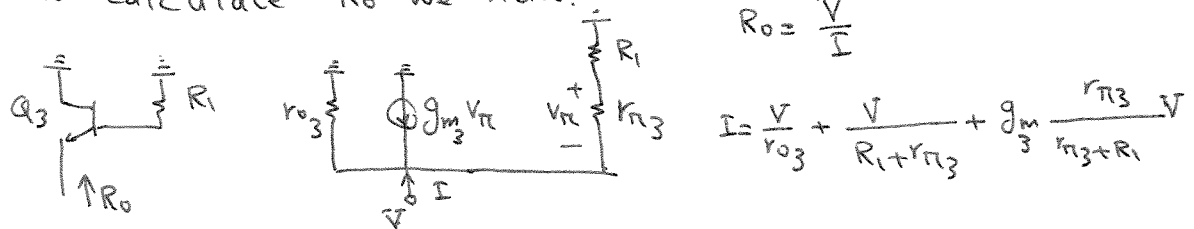
(c)



half circuit



To calculate R_o we have:

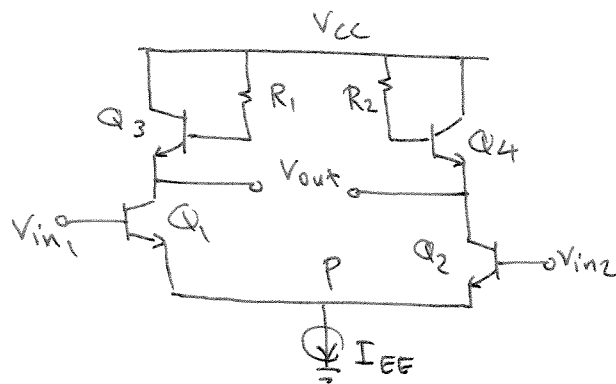


$$\Rightarrow R_o = r_{o3} \parallel (R_1 + r_{\pi 3}) \parallel (1 + \frac{R_1}{r_{\pi 3}}) \frac{1}{g_{m3}}$$

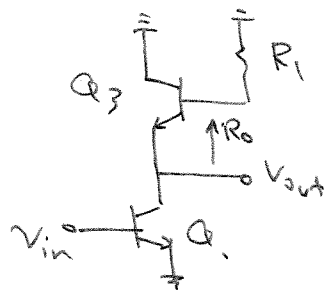
$$A_v = \frac{V_{out}}{V_{in}} = \frac{V_x}{V_{in}} \frac{V_{out}}{V_x} = -g_{m1} (r_{o1} \parallel R_o) \frac{R_1}{R_1 + r_{\pi 3}}$$

(32)

(d)



From half circuit concept :

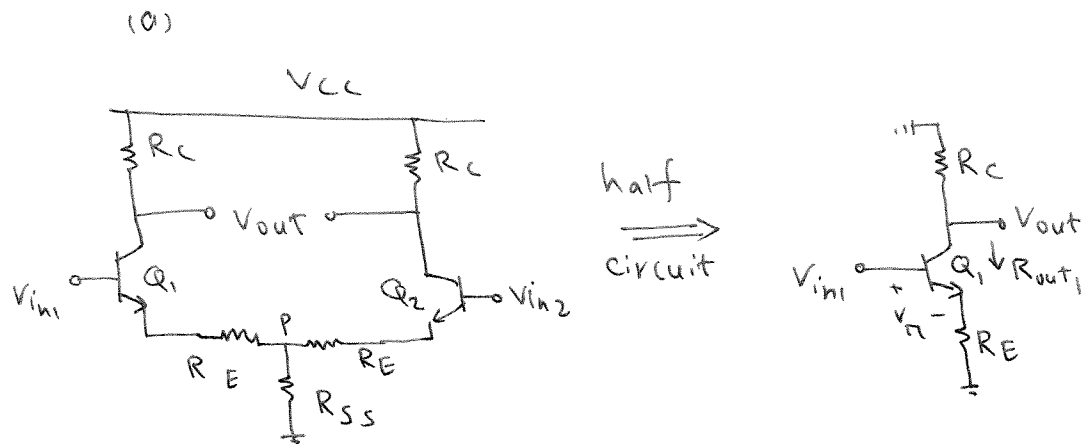


we already proved in part (c) that

$$R_o = r_{o3} \parallel (R_1 + r_{\pi 3}) \parallel \left(1 + \frac{R_1}{r_{\pi 3}}\right) \frac{1}{g_{m3}}$$

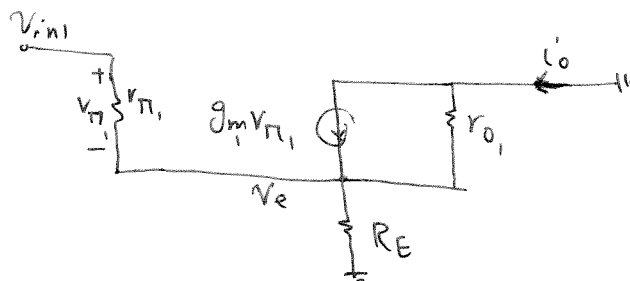
$$\rightarrow A_v = \frac{v_{out}}{v_{in}} = -g_{m1} (r_{o1} \parallel R_o)$$

(33)



$$R_{out} = R_C \parallel R_{out1} = R_C \parallel (g_{m1} r_{o1} (R_E \parallel r_{\pi1}) + r_{o1} + (R_E \parallel r_{\pi1}))$$

To calculate G_m , the small signal model is:



writing node equation at node v_e :

$$\frac{v_e}{R_E \parallel r_{o1}} = (g_{m1} + \frac{1}{r_{\pi1}}) v_{\pi1} = \underbrace{(g_{m1} + \frac{1}{r_{\pi1}})}_{g_{m1}} (V_{in1} - v_e)$$

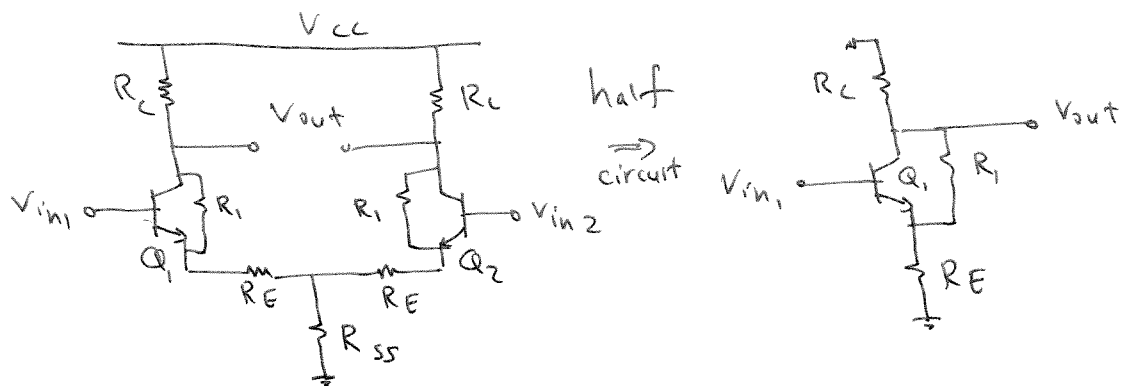
$$\Rightarrow v_e = \frac{g_{m1}}{g_{m1} + \frac{1}{R_E \parallel r_{o1}}} V_{in1} \Rightarrow i_o = \frac{v_e}{r_{o1}} - g_{m1} v_{\pi1}$$

$$= \frac{v_e}{r_{o1}} + g_{m1} (v_e - V_{in1}) \Rightarrow G_m = \frac{i_o}{V_{in1}} = + \frac{g_{m1} r_{o1}}{g_{m1} r_{o1} R_E + r_{o1} + R_E}$$

$$\Rightarrow A_v = -G_m R_{out}$$

(33)

(b)



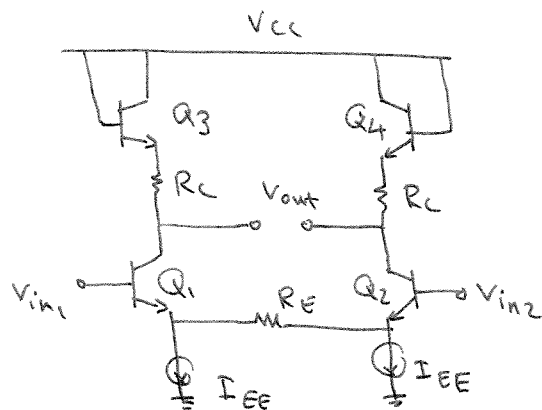
$$R_{out} = R_C \parallel \left(g_{m_1} (r_{o_1} \parallel R_1) (R_E \parallel r_{m_1}) + (r_{o_1} \parallel R_1) + (R_E \parallel r_{m_1}) \right)$$

Similar to the approach in part (a)

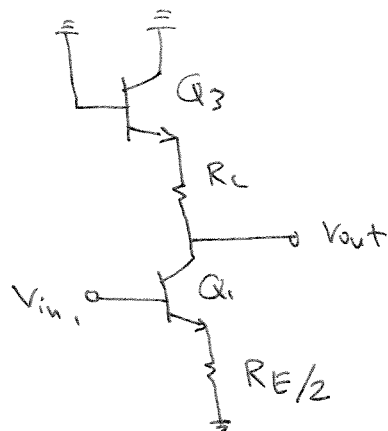
$$G_m = + \frac{g_{m_1} (r_{o_1} \parallel R_1)}{g_{m_1} (r_{o_1} \parallel R_1) R_E + (r_{o_1} \parallel R_1) + R_E}$$

$$\Rightarrow A_v = -G_m R_{out}$$

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The half circuit is shown as:

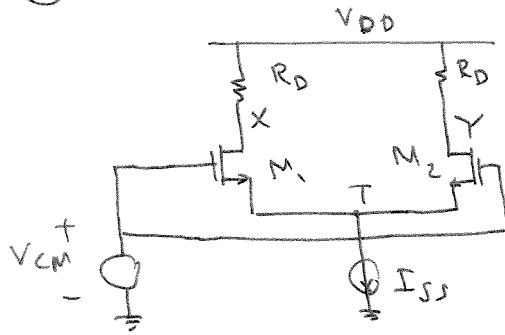


$$a) \quad A_v = \frac{v_{out}}{v_{in1}} = - \frac{R_C + 1/g_{m3}}{R_{E/2} + 1/g_{m1}}$$

$$b) \quad \text{if } \frac{R_C}{R_{E/2}} = A, \text{ then if } \frac{1/g_{m3}}{1/g_{m1}} = A$$

we conclude $A_v = -A$. So the circuit is very linear.

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$$V_T = V_{CM} - V_{GS1} = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

(a) $V_T = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{2W}{L}}}$

The tail voltage increases

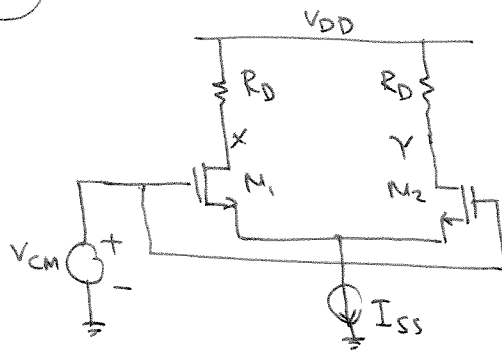
(b) $V_T = V_{CM} - V_{TH} - \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$

The tail voltage decreases

(c) $V_T = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n \frac{C_{ox}}{2} \frac{W}{L}}}$

The tail voltage decreases

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$$\begin{aligned} V_{CM} &= 1V \\ I_{SS} &= 1mA \\ R_D &= 1k\Omega \end{aligned}$$

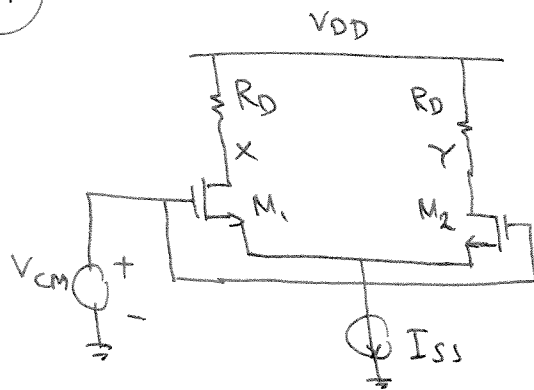
$$V_X = V_{DD} - R_D \frac{I_{SS}}{2} = V_{DD} - 0.5$$

To ensure that the devices work in saturation

$$V_{CM} \leq V_X - V_{TH} \rightarrow V_{DD} - 0.5 - V_{TH} \geq 1$$

$$\text{if } V_{TH} = 0.5 \rightarrow V_{DD} > 2$$

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$$V_{GS} - V_{TH} = 200 \text{ mV}$$

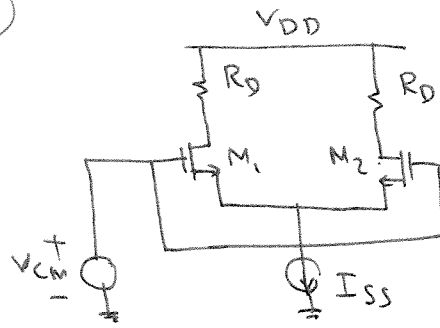
$$\mu_n C_{ox} = 100 \text{ } \mu\text{A/V}^2$$

$$\frac{W}{L} = 20/0.18$$

$$(V_{GS} - V_{TH})_{\text{equil}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \rightarrow$$

$$0.2 = \sqrt{\frac{I_{SS}}{10^{-4} \times \frac{20}{0.18}}} \rightarrow I_{SS} = 0.44 \text{ mA}$$

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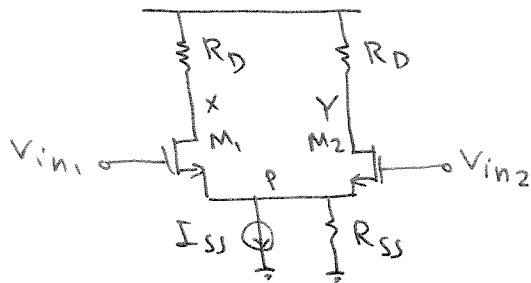
$$\frac{I_{D1}}{W} = \frac{I_{SS}}{2W} = J$$

current density

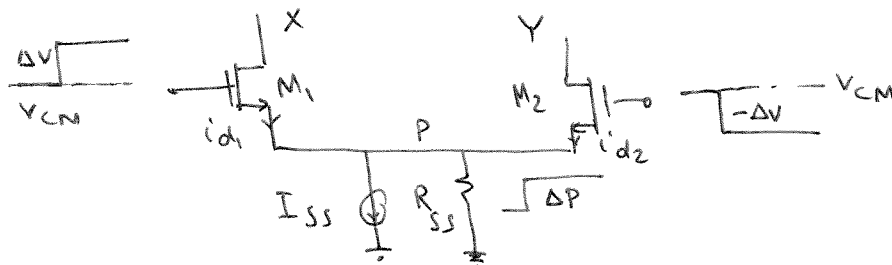
$$(V_{GS} - V_{TH})_{\text{equil}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} =$$

$$\sqrt{\frac{2J}{\mu_n C_{ox} \frac{1}{L}}}$$

(39)



$$g_{m1} = g_{m2} = g_m$$



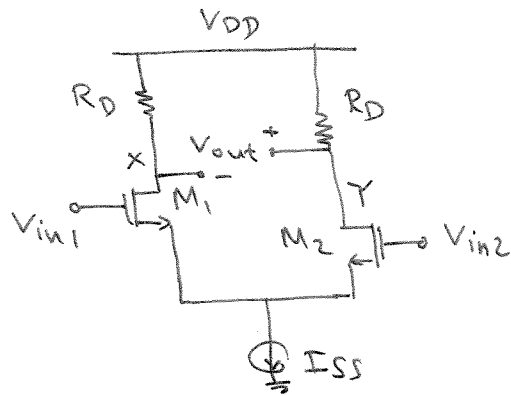
if V_{in1} and V_{in2} change by ΔV and $-\Delta V$ we have:

node equation at P: $i_{d1} + i_{d2} + \frac{\Delta P}{R_{SS}} = 0 \Rightarrow$

$$g_m (\Delta V - \Delta P) + g_m (-\Delta V - \Delta P) + \frac{\Delta P}{R_{SS}} = 0 \Rightarrow$$

$$-g_m \Delta P - g_m \Delta P + \frac{\Delta P}{R_{SS}} = 0 \Rightarrow \Delta P = 0$$

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$$V_{in1} = 1.5$$
$$V_{in2} = 0.3$$

$$V_x - V_{TH} > V_{in1} \rightarrow V_{DD} - R_D I_{SS} - V_{TH} > 1.5$$

41

$$P = 2 \text{ mW}$$

$$V_{DD} = 2 \text{ V} \rightarrow I_{SS} = 1 \text{ mA}$$

$$V_{CM, out} = V_{DD} - R_D \frac{I_{SS}}{2} = 2 - 0.5 R_D \rightarrow$$

$$R_D = \frac{2 - V_{CM, out}}{0.5} \text{ (k}\Omega\text{)}$$

$$g_m R_D = 5 \rightarrow \sqrt{2 \mu_n C_{ox} \frac{W}{L} \frac{I_{SS}}{2}} \cdot \frac{2 - V_{CM, out}}{0.5} = 5 \quad \downarrow \text{k}\Omega$$

$$\rightarrow \sqrt{2 \times 10^{-4} \frac{W}{L} \frac{10^{-3}}{2}} \cdot 2 \times 10^3 (2 - V_{CM, out}) = 5$$

$$\text{if } V_{CM, out} = 1.6 \rightarrow \frac{W}{L} = 390.625$$

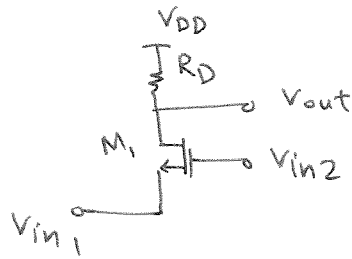
To formulate the trade off we have:

$$I_{SS} = \frac{2}{V_{DD}} \text{ mA} \Rightarrow R_D = \frac{V_{DD} - V_{CM, out}}{1} \times V_{DD} \text{ (k}\Omega\text{)}$$

$$\Rightarrow |g_m R_D| = 5 \Rightarrow$$

$$\sqrt{2 \times 10^{-4} \left(\frac{W}{L}\right) \frac{1}{V_{DD}} \times 10^{-3}} (V_{DD} - V_{CM, out}) V_{DD} = 5$$

(42)



$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right) (V_{in2} - V_{in1} - V_{TH})^2$$

- (1) The current is not an odd function of $(V_{in2} - V_{in1})$. Therefore it is not symmetric around $V_{in1} = V_{in2} [(V_{in1} - V_{in2}) = 0]$.
- (2) The input impedance seen at V_{in1} and V_{in2} are different
- (3) The circuit cannot suppress the supply noise, because there is no differential output available.

(43)

$$(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - 2\sqrt{I_{D1}I_{D2}})$$

(a)

$$I_{D1} = 0 \Rightarrow$$

$$(V_{in1} - V_{in2})^2 = \frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}} \rightarrow V_{in1} - V_{in2} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

This is the minimum differential input voltage to turn M_1 off.

$$(b) \quad I_{D1} = \frac{I_{SS}}{2} \Rightarrow I_{D2} = \frac{I_{SS}}{2}$$

$$(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - I_{SS}) = 0 \rightarrow V_{in1} - V_{in2} = 0$$

This is the equilibrium input case.

$$(c) \quad I_{D1} = I_{SS} \rightarrow I_{D2} = 0$$

$$(V_{in1} - V_{in2})^2 = \frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}} \rightarrow V_{in1} - V_{in2} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

This is the minimum input differential

voltage to turn M_2 off.

(44)

$$I_{D1} = \frac{I_{SS}}{2} - \frac{1}{4} \sqrt{4I_{SS}^2 - \left[\mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 - 2I_{SS} \right]}$$

The analyses which led to the above equation assume that the transistors work in saturation region.

So,

$$-(V_{in1} - V_{in2})_{\max} \leq V_{in1} - V_{in2} \leq (V_{in1} - V_{in2})_{\max}$$

$$(V_{in1} - V_{in2})_{\max} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$\mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 \leq 2I_{SS} \Rightarrow$$

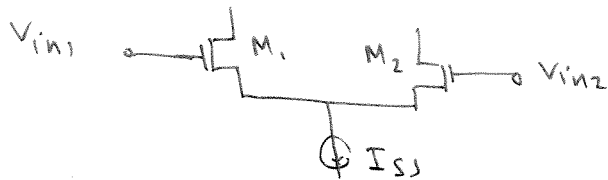
$$- \left[\mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 - 2I_{SS} \right] \geq 0 \Rightarrow$$

$$\frac{1}{4} \sqrt{4I_{SS}^2 - \left[\mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 - 2I_{SS} \right]} \geq \frac{1}{2} I_{SS}$$

$$\Rightarrow I_{D1} < 0$$

(45)

$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$$



The equilibrium overdrive voltage is:

$$(V_{GS1} - V_{TH})_{\text{equil}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = V_{OV} \Rightarrow$$

$$\mu_n C_{ox} \frac{W}{L} = \frac{I_{SS}}{V_{OV}^2} \quad \text{therefore}$$

$$I_{D1} - I_{D2} = \frac{I_{SS}}{2} \frac{(V_{in1} - V_{in2})}{V_{OV}^2} \sqrt{\frac{4I_{SS}}{\frac{I_{SS}}{V_{OV}^2}} - (V_{in1} - V_{in2})^2} \Rightarrow$$

$$I_{D1} - I_{D2} = I_{SS} \Rightarrow$$

$$I_{SS} = \frac{I_{SS}}{2} \frac{(V_{in1} - V_{in2})}{V_{OV}^2} \sqrt{4V_{OV}^2 - (V_{in1} - V_{in2})^2}$$

$$\Rightarrow (V_{in1} - V_{in2})^4 - 4V_{OV}^2 (V_{in1} - V_{in2})^2 + 4V_{OV}^4 = 0$$

$$\Rightarrow ((V_{in1} - V_{in2})^2 - 2V_{OV}^2)^2 = 0 \Rightarrow$$

$$V_{in1} - V_{in2} = \sqrt{2} V_{OV} = \sqrt{2} \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

(46)

$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

$$V_{ov} = (V_{GS} - V_{TH})_{\text{equil}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow \mu_n C_{ox} \frac{W}{L} = \frac{I_{SS}}{V_{ov}^2}$$

$$\Rightarrow I_{D1} - I_{D2} = \frac{I_{SS}}{2} \frac{(V_{in1} - V_{in2})}{V_{ov}^2} \sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2}$$

$$\Rightarrow G_m = \frac{\partial(I_{D1} - I_{D2})}{\partial(V_{in1} - V_{in2})} =$$

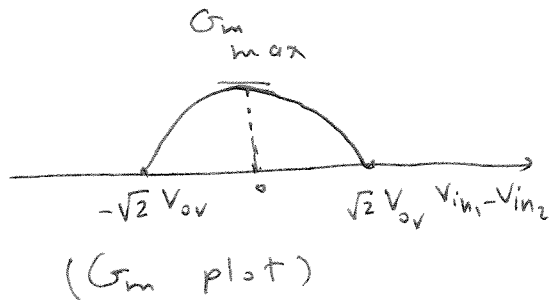
$$\frac{I_{SS}}{2V_{ov}^2} \left[\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2} - \frac{(V_{in1} - V_{in2})^2}{\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2}} \right] =$$

$$\frac{I_{SS}}{2V_{ov}^2} \frac{4V_{ov}^2 - 2(V_{in1} - V_{in2})^2}{\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2}} =$$

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - 2(V_{in1} - V_{in2})^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}}$$

$$V_{in1} - V_{in2} = 0 \Rightarrow$$

$$G_{m_{\max}} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}$$



47

From problem 46:

$$G_{m_{\max}} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} = \sqrt{\frac{I_{SS}}{V_{ov}^2} I_{SS}} = \frac{I_{SS}}{V_{ov}}$$

$$\Rightarrow \text{if } G_m = \frac{1}{2} \frac{I_{SS}}{V_{ov}} \text{ we have}$$

$$\frac{1}{2} \frac{I_{SS}}{V_{ov}} = \frac{I_{SS}}{2 V_{ov}^2} \frac{4V_{ov}^2 - 2(V_{in1} - V_{in2})^2}{\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2}} \Rightarrow$$

$$V_{ov} = \frac{4V_{ov}^2 - 2(V_{in1} - V_{in2})^2}{\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2}} \Rightarrow$$

$$(4V_{ov}^2 - (V_{in1} - V_{in2})^2) V_{ov}^2 = 16V_{ov}^4 + 4(V_{in1} - V_{in2})^4 - 16V_{ov}^2 (V_{in1} - V_{in2})^2$$

$$\Rightarrow 4(V_{in1} - V_{in2})^4 - 15V_{ov}^2 (V_{in1} - V_{in2})^2 + 12V_{ov}^4 = 0$$

$$\Rightarrow (V_{in1} - V_{in2})^2 = \frac{15V_{ov}^2 \pm \sqrt{225V_{ov}^4 - 192V_{ov}^4}}{8} =$$

$$\frac{15V_{ov}^2 \pm \sqrt{33} V_{ov}^2}{8}$$

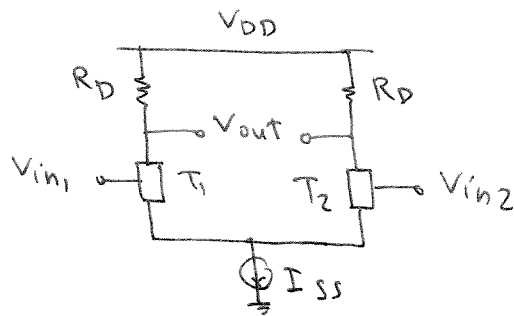
positive sign is not accepted

$$\text{because } (V_{in1} - V_{in2})^2 \leq 2V_{ov}^2 \Rightarrow$$

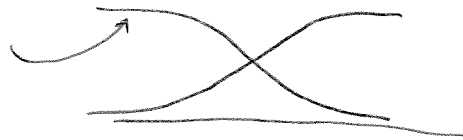
$$V_{in1} - V_{in2} = \pm \sqrt{\frac{15 - \sqrt{33}}{8}} V_{ov} = \pm 1.0756 V_{ov}$$

48-

$$I_D = \gamma (V_{GS} - V_{TH})^3$$



(a) The characteristic of $I_{D1} - I_{D2}$ vs. $V_{in1} - V_{in2}$ is similar to the standard CMOS differential pair. because it has saturation part.



(b) $I_D = \frac{I_{SS}}{2} = \gamma (V_{GS} - V_{TH})^3 \Rightarrow$

$$(V_{GS} - V_{TH})_{\text{equil}} = \sqrt[3]{\frac{I_{SS}}{2\gamma}}$$

(c) $I_{D1} = I_{SS} = \gamma (V_{GS1} - V_{TH})^3 \Rightarrow V_{GS1} - V_{TH} = \sqrt[3]{\frac{I_{SS}}{\gamma}}$

$I_{D2} = 0 = \gamma (V_{GS2} - V_{TH})^3 \Rightarrow V_{GS2} - V_{TH} = 0$

$\Rightarrow V_{GS1} - V_{GS2} = V_{in1} - V_{in2} = \sqrt[3]{\frac{I_{SS}}{\gamma}} =$

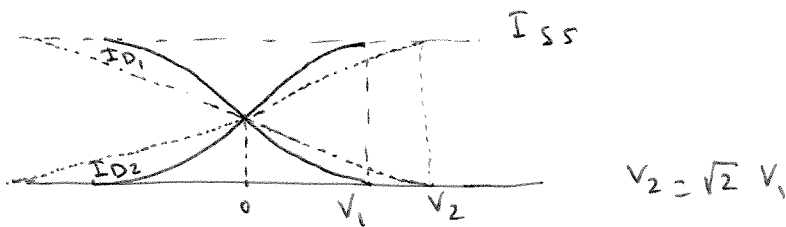
$$\sqrt[3]{2} (V_{GS} - V_{TH})_{\text{equil}}$$

(49)

(a)

gate oxide thickness is doubled $\Rightarrow C_{ox}$ is halved $\Rightarrow$

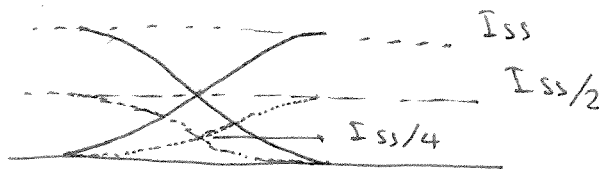
$(V_{in1} - V_{in2})_{max}$ scales up by $\sqrt{2}$.



so all the curves stretch out to the sides by $\sqrt{2}$ times.

(b) if threshold voltage is halved, nothing will change in the curves. The reason is that the curves depend on $V_{in1} - V_{in2}$.

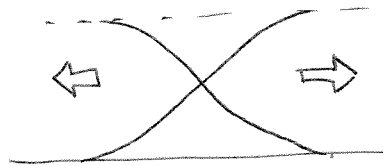
(c) In this case, $(V_{in1} - V_{in2})_{max}$ does not change so all the curves scale half downward because I_{SS} is halved.



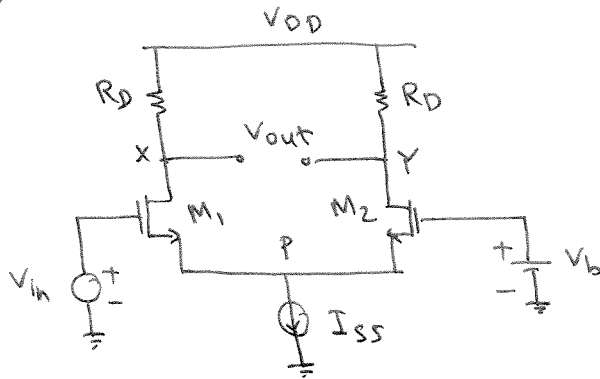
(50)

if mobility falls then $(V_{in1} - V_{in2})_{max}$ will increase because $(V_{in1} - V_{in2})_{max} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$

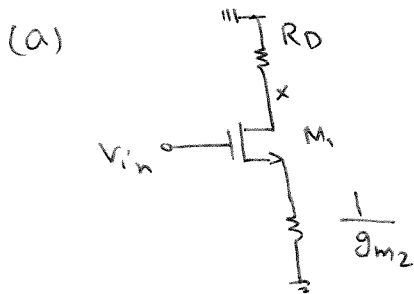
So the curves stretch out to the sides.



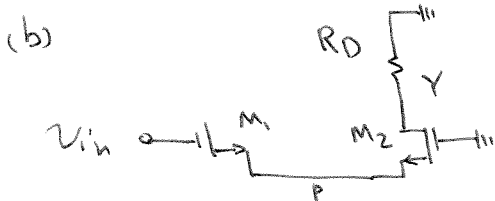
(51)



$$g_{m1} = g_{m2} = g_m$$



$$\begin{aligned} v_x &= -g_{m1} v_{gs1} R_D = \\ &= -g_{m1} \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in} R_D = \\ &= -\frac{g_{m1} g_{m2}}{g_{m1} + g_{m2}} R_D v_{in} = -\frac{g_m}{2} R_D v_{in} \end{aligned}$$



$$v_p = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in}$$

$$\Rightarrow v_p = \frac{g_{m1}}{g_{m1} + g_{m2}} v_{in} \Rightarrow$$

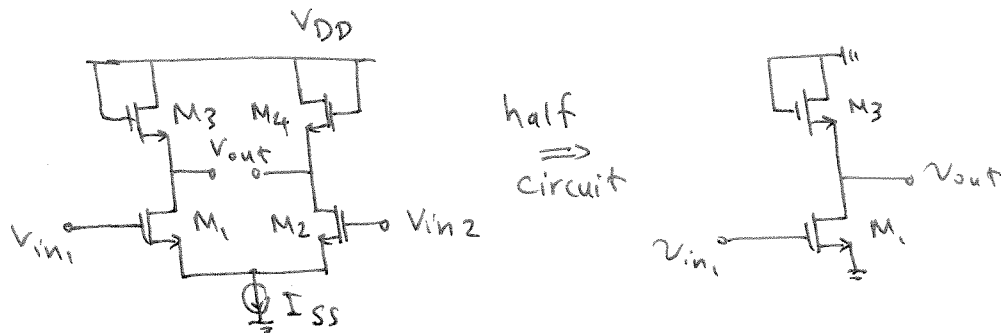
$$v_y = -g_{m2} v_{gs2} R_D = g_{m2} v_p R_D = \frac{g_{m1} g_{m2}}{g_{m1} + g_{m2}} R_D v_{in}$$

$$\Rightarrow v_y = \frac{g_m}{2} R_D v_{in}$$

(c) $\frac{v_x - v_y}{v_{in}} = -g_m R_D$ This value is equal to the gain of the differential amplifier.

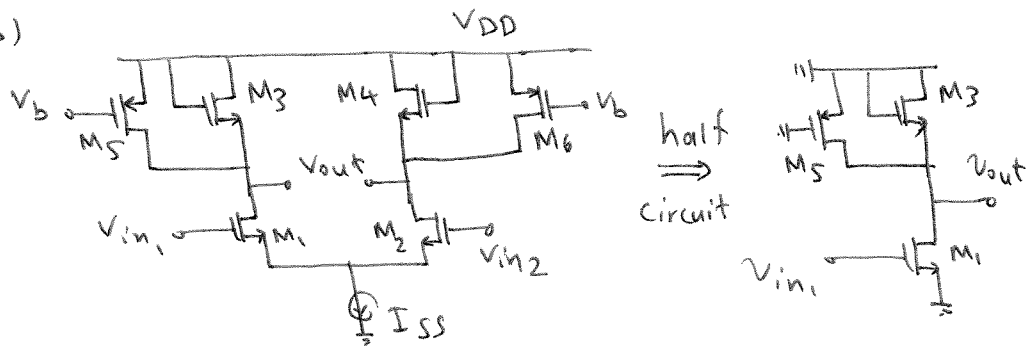
(52)

(a)



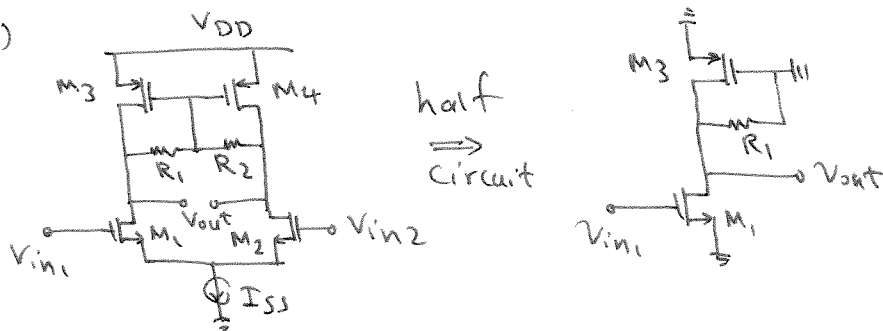
$$A_v = -g_{m1} (r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{m3}})$$

(b)



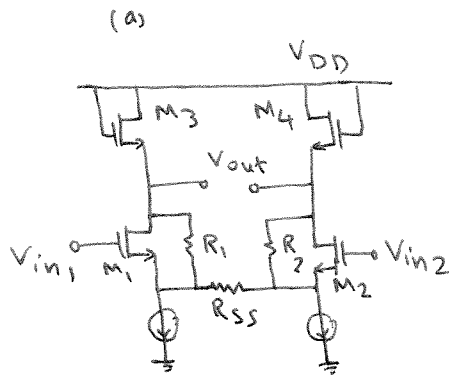
$$A_v = -g_{m1} (r_{o1} \parallel r_{o5} \parallel \frac{1}{g_{m3}} \parallel r_{o3})$$

(c)

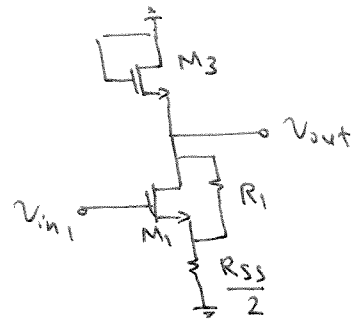


$$A_v = -g_{m1} (r_{o1} \parallel r_{o3} \parallel R_1)$$

(53)

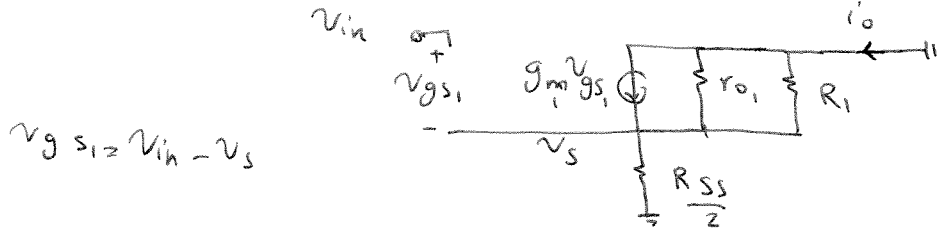


half
 $\Rightarrow$
 circuit



$$R_{out} = (r_{o3} \parallel \frac{1}{g_{m3}}) \parallel (g_{m1} (R_1 \parallel r_{o1}) \frac{R_{SS}}{2} + \frac{R_{SS}}{2} + R_1 \parallel r_{o1})$$

To calculate G_m :



$$\frac{v_s}{\frac{R_{SS}}{2} \parallel R_1 \parallel r_{o1}} + g_{m1} v_s = g_{m1} v_{in} \Rightarrow v_s = \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel R_1 \parallel r_{o1}}}$$

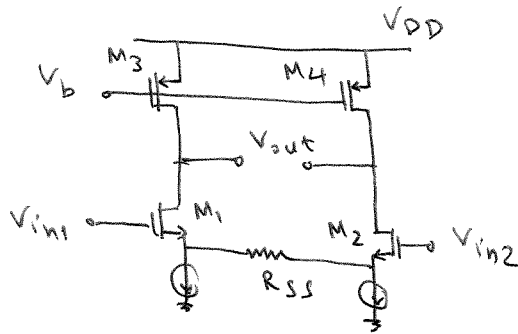
$$i_o = + \frac{v_s}{\frac{R_{SS}}{2}} = + \frac{1}{\frac{R_{SS}}{2}} \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel R_1 \parallel r_{o1}}} \Rightarrow$$

$$G_m = \frac{i_o}{v_{in}} = + \frac{2 g_{m1}}{R_{SS}} \frac{1}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel R_1 \parallel r_{o1}}}$$

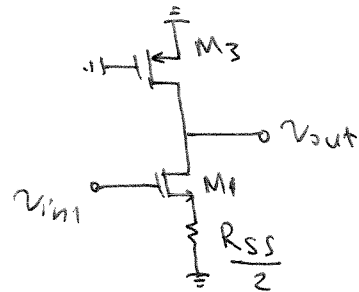
$$A_v = -G_m R_{out}$$

53

(b)

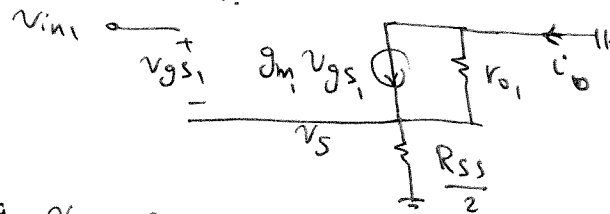


half
⇒
circuit



$$R_{out} = r_{o3} \parallel \left(g_{m1} r_{o1} \frac{R_{SS}}{2} + r_{o1} + \frac{R_{SS}}{2} \right)$$

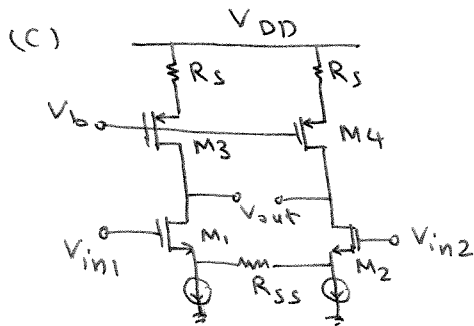
To calculate G_m :



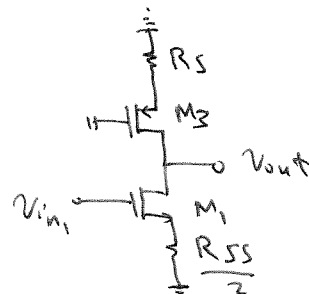
$$\frac{v_s}{r_{o1} \parallel \frac{R_{SS}}{2}} + g_{m1} v_s = g_{m1} v_{in} \Rightarrow v_s = \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel r_{o1}}}$$

$$G_m = \frac{i_o}{v_{in}} = + \frac{v_s}{\frac{R_{SS}}{2}} \frac{1}{v_{in}} = + \frac{2g_{m1}}{R_{SS}} \frac{1}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel r_{o1}}}$$

$$\rightarrow A_{v3} = -G_m R_{out}$$



half
⇒
Circuit

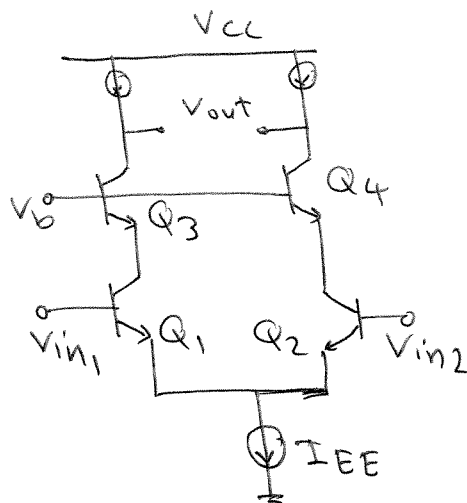


$$R_{out} = (g_{m3} r_{o3} R_S + r_{o3} + R_S) \parallel \left(g_{m1} r_{o1} \frac{R_{SS}}{2} + r_{o1} + \frac{R_{SS}}{2} \right)$$

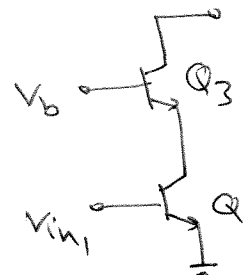
G_m for this circuit is equal to the one for part (b) so:

$$G_m = + \frac{2g_{m1}}{R_{SS}} \frac{1}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \parallel r_{o1}}} \Rightarrow A_{v2} = -G_m R_{out}$$

(54)



half
 $\Rightarrow$
 circuit



$$A_v = 4000$$

$$\beta = 100$$

$$A_v = -g_{m1} [g_{m3}(r_{o1} \parallel r_{\pi 3})r_{o3} + r_{o3} + r_{o1} \parallel r_{\pi 3}]$$

$$g_{m1-4} = \frac{I_{EE}}{2V_T} \quad r_{o1-4} = \frac{2V_A}{I_{EE}} \quad r_{\pi 3} = \frac{2V_T \beta}{I_{EE}}$$

$$4000 = \frac{I_{EE}}{2V_T} \left[\frac{I_{EE}}{2V_T} \left(\frac{2V_A}{I_{EE}} \parallel \frac{2V_T \beta}{I_{EE}} \right) \frac{2V_A}{I_{EE}} + \frac{2V_A}{I_{EE}} + \left(\frac{2V_A}{I_{EE}} \parallel \frac{2V_T \beta}{I_{EE}} \right) \right]$$

$$\Rightarrow 4000 = \frac{1}{2V_T} \left[\frac{V_A}{V_T} (2V_A \parallel 2V_T \beta) + 2V_A + (2V_A \parallel 2V_T \beta) \right] \Rightarrow$$

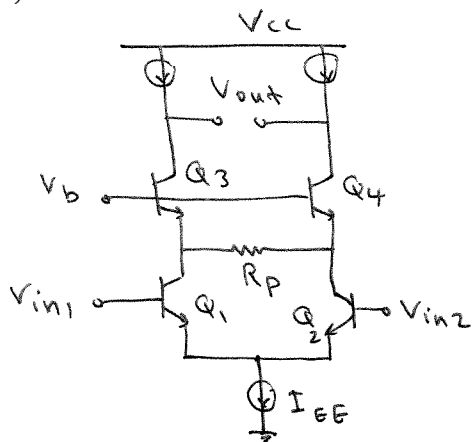
$$4000 = \frac{1}{V_T} \left[\frac{V_A}{V_T} (V_A \parallel \beta V_T) + V_A + (V_A \parallel \beta V_T) \right] \Rightarrow$$

$$4000 = \frac{1}{V_T} \left[\frac{\beta V_A^2}{\beta V_T + V_A} + V_A + \frac{\beta V_A V_T}{\beta V_T + V_A} \right] \Rightarrow$$

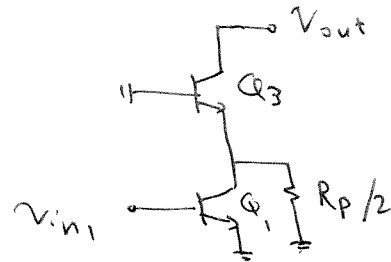
$$4000 = \frac{1}{0.026} \left[\frac{100 V_A^2}{2.6 + V_A} + V_A + \frac{2.6 V_A}{2.6 + V_A} \right] \Rightarrow$$

$$V_A = 2.197$$

(55)



half
⇒
circuit

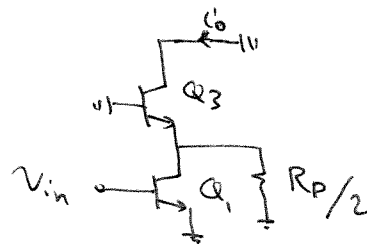


$$R_{out} = g_{m3} r_{o3} \left(\frac{R_p \parallel r_{o1} \parallel r_{o3}}{2} \right) + r_{o3} + \frac{R_p \parallel r_{o1} \parallel r_{o3}}{2}$$

To calculate G_m

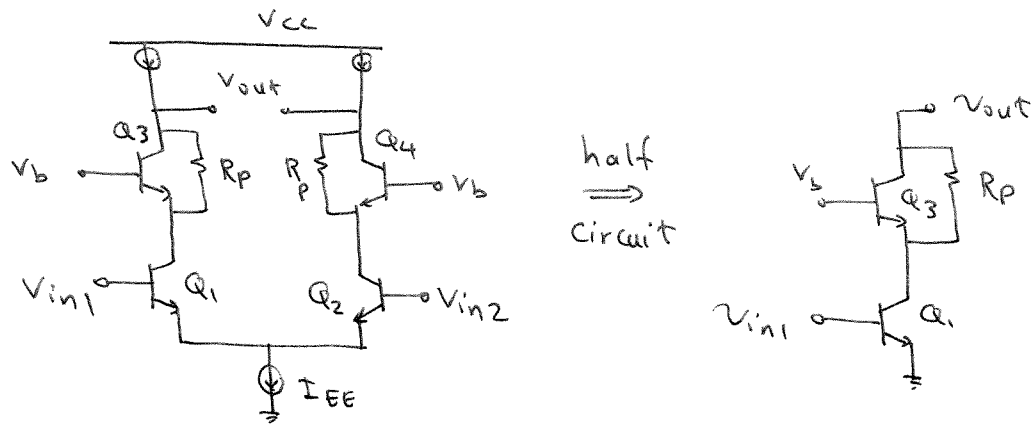
$$G_m = \frac{i_o}{v_{in}}$$

$$\frac{i_o}{v_{in}} = +g_{m1} \frac{g_{m3} \left(\frac{R_p \parallel r_{o1}}{2} \right)}{g_{m3} \left(\frac{R_p \parallel r_{o1}}{2} \right) + 1}$$



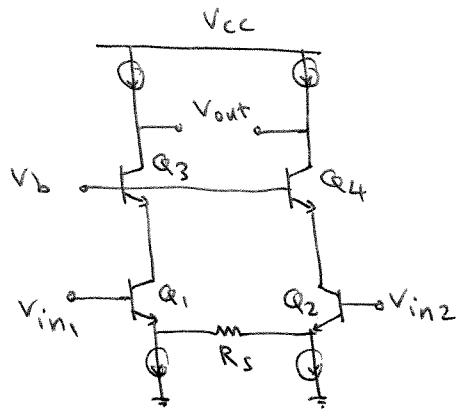
$$A_v = -G_m R_{out}$$

(56)

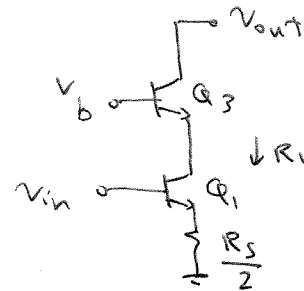


$$A_v = -g_{m_1} (g_{m_3} (r_{o_3} \parallel R_P) (r_{o_1} \parallel r_{\pi_3}) + (r_{o_3} \parallel R_P) + (r_{o_1} \parallel r_{\pi_3}))$$

(57)



half
⇒
circuit



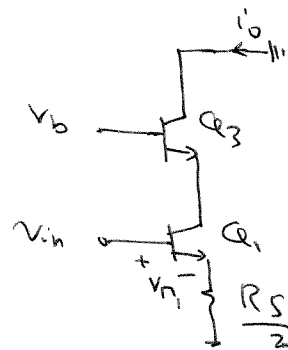
$$R_1 = g_{m1} r_{o1} \left(\frac{R_S}{2} \parallel r_{n1} \right) + r_{o1} + \frac{R_S}{2} \parallel r_{n1}$$

$$R_{out} = g_{m3} r_{o3} (R_1 \parallel r_{n3}) + r_{o3} + (R_1 \parallel r_{n3})$$

To calculate G_m :

$$v_{n1} \approx \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{R_S}{2}} v_{in}$$

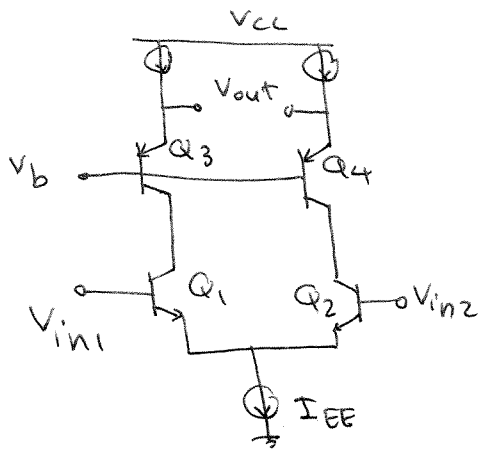
$$= \frac{1}{1 + g_{m1} \frac{R_S}{2}} v_{in}$$



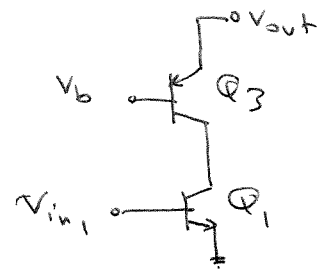
$$G_m = \frac{i_o}{v_{in}} = \frac{+g_{m1} v_{n1}}{v_{in}} = \frac{+g_{m1}}{1 + g_{m1} \frac{R_S}{2}}$$

$$A_v = -G_m R_{out}$$

(58)

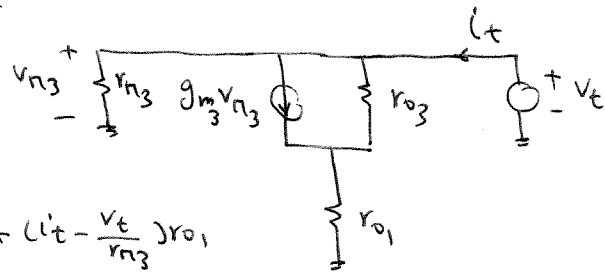


half
 $\Rightarrow$
 circuit



To calculate R_{out}

$$v_{n3} = v_t$$



$$v_t = (i_t - g_{m3} v_t - \frac{v_t}{r_{n3}}) r_{o3} + (i_t - \frac{v_t}{r_{n3}}) r_{o1}$$

$$\rightarrow v_t \left(1 + g_{m3} r_{o3} + \frac{r_{o3}}{r_{n3}} + \frac{r_{o1}}{r_{n3}} \right) = i_t (r_{o1} + r_{o3})$$

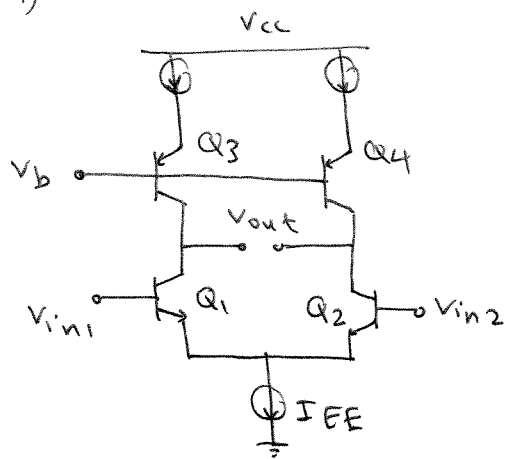
$$\rightarrow \frac{v_t}{i_t} = \frac{r_{o1} + r_{o3}}{1 + g_{m3} r_{o3} + \frac{g_{m3} r_{o3}}{\beta_3} + \frac{g_{m3} r_{o1}}{\beta_3}} = R_{out} \rightarrow$$

$$R_{out} \approx \frac{r_{o1} + r_{o3}}{g_{m3} r_{o3}}$$

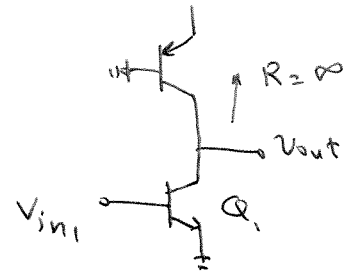
$$G_m = +g_{m1} \Rightarrow$$

$$A_v = -G_m R_{out} = -g_{m1} \frac{r_{o1} + r_{o3}}{g_{m3} r_{o3}}$$

(59)

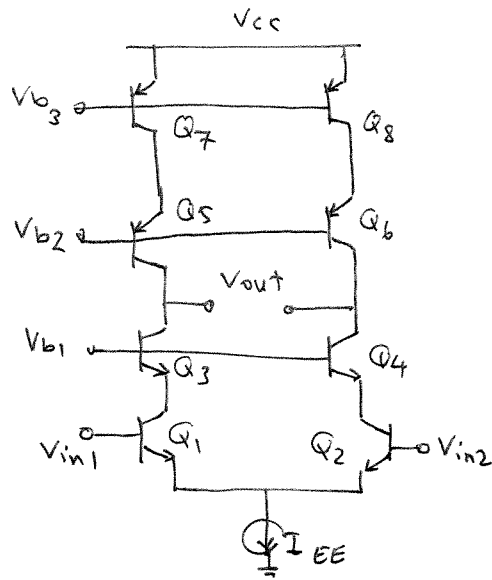


half
 $\Rightarrow$
circuit

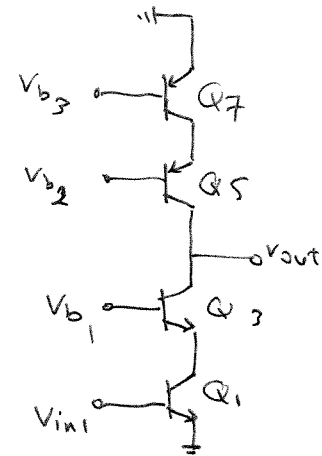


$$A_v = -g_{m1} r_{o1}$$

60



half circuit
 $\Rightarrow$



$$A_v = 800, \quad \beta_n = 2\beta_p = 100, \quad V_{A,n} = 2V_{A,p}$$

$$A_v \approx -g_{m1} \left[g_{m3} r_{o3} (r_{o1} \parallel r_{\pi3}) \right] \parallel \left[g_{m5} r_{o5} (r_{o7} \parallel r_{\pi5}) \right] =$$

$$- \frac{I_{EE}}{2V_T} \left[\frac{I_{EE}}{2V_T} \frac{V_{A,n}}{\frac{I_{EE}}{2}} \left(\frac{V_{A,n}}{\frac{I_{EE}}{2}} \parallel \frac{\beta_n V_T}{\frac{I_{EE}}{2}} \right) \right] \parallel \left[\frac{I_{EE}}{2V_T} \frac{V_{A,p}}{\frac{I_{EE}}{2}} \left(\frac{V_{A,p}}{\frac{I_{EE}}{2}} \parallel \frac{\beta_p V_T}{\frac{I_{EE}}{2}} \right) \right]$$

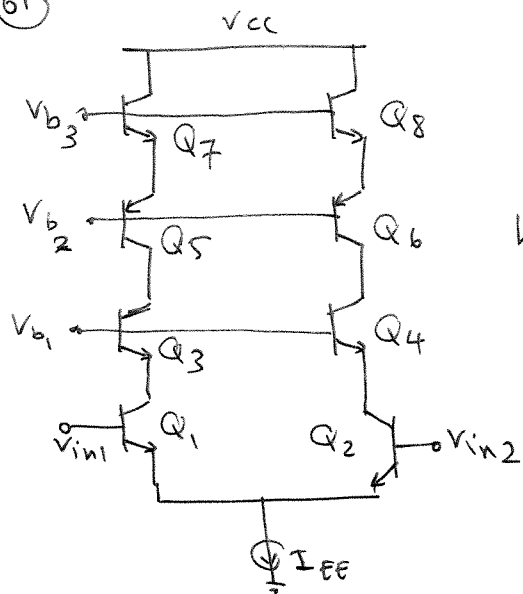
$$= - \frac{1}{V_T^2} \left[V_{A,n} (V_{A,n} \parallel \beta_n V_T) \right] \parallel \left[V_{A,p} (V_{A,p} \parallel \beta_p V_T) \right] =$$

$$- \frac{1}{V_T^2} \left[V_{A,n} (V_{A,n} \parallel \beta_n V_T) \right] \parallel \left[\frac{V_{A,n}}{2} \left(- \frac{V_{A,n}}{2} \parallel \frac{\beta_n}{2} V_T \right) \right] \Rightarrow$$

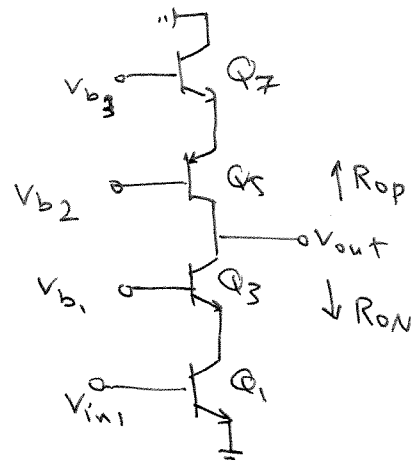
$$800 = \frac{1}{5V_T^2} (V_{A,n} (V_{A,n} \parallel 2.6)) \Rightarrow V_{A,n} = 2.245 \text{ V}$$

$$\Rightarrow V_{A,p} = 1.225 \text{ V}$$

61



half circuit
 $\Rightarrow$



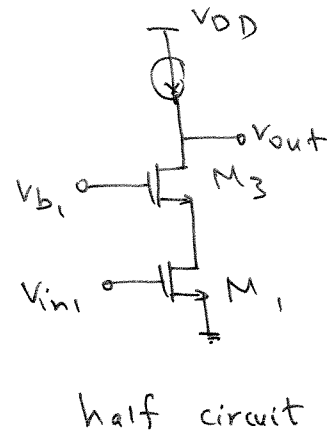
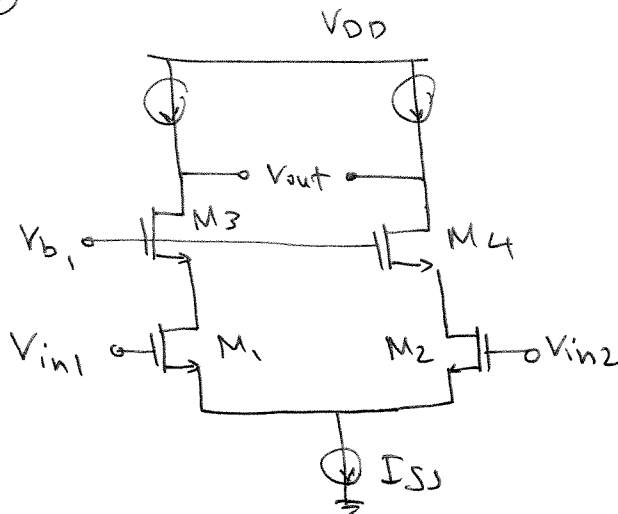
$$R_{ON} = (r_{o1} \parallel r_{\pi 3}) + r_{o3} + g_{m3} r_{o3} (r_{o1} \parallel r_{\pi 3})$$

$$R_{OP} = \frac{1}{g_{m7}} \parallel r_{o7} \parallel r_{\pi 5} + r_{o5} + g_{m5} r_{o5} \left(\frac{1}{g_{m7}} \parallel r_{o7} \parallel r_{\pi 5} \right) \approx$$

$$\frac{1}{g_{m7}} \parallel r_{\pi 5} + r_{o5} + g_{m5} r_{o5} \left(\frac{1}{g_{m7}} \parallel r_{\pi 5} \right)$$

$$A_v = -g_{m1} (R_{ON} \parallel R_{OP})$$

(62)



$$A_v = 300, \quad W/L = \frac{20}{0.18}, \quad \mu_n C_{ox} = 100 \mu A/V^2$$

$$\lambda = 0.1 V^{-1}$$

$$A_v \approx -g_{m3} r_{o3} g_{m1} r_{o1}$$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \frac{I_{SS}}{2}}, \quad g_{m3} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_3 \frac{I_{SS}}{2}}$$

$$\rightarrow g_{m1} = g_{m3} = \sqrt{10^{-4} \frac{20}{0.18} I_{SS}}$$

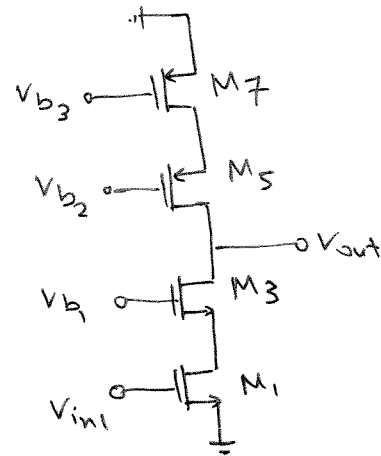
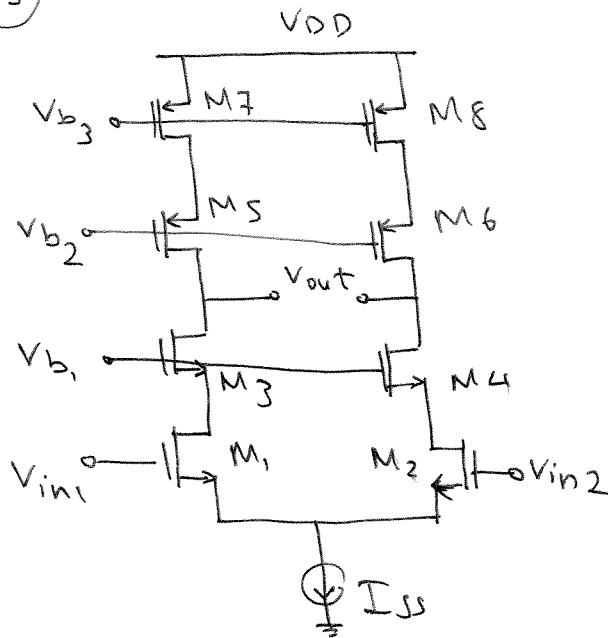
$$r_{o1} = \frac{1}{\lambda \frac{I_{SS}}{2}}, \quad r_{o3} = \frac{1}{\lambda \frac{I_{SS}}{2}} \rightarrow r_{o1} = r_{o3} = \frac{20}{I_{SS}}$$

So:

$$300 = \left(10^{-4} \frac{20}{0.18} I_{SS}\right) \frac{400}{2 I_{SS}} \Rightarrow$$

$$I_{SS} = 14.815 \text{ mA}$$

63



$A_v = 200$, $I_{SS} = 1\text{mA}$, $\mu_n C_{ox} = 100 \mu\text{A/V}^2$
 $\mu_p C_{ox} = 50 \mu\text{A/V}^2$, $\lambda_n = 0.1\text{V}^{-1}$, $\lambda_p = 0.2\text{V}^{-1}$
 $(\frac{W}{L})_1 = \dots = (\frac{W}{L})_8 = ?$

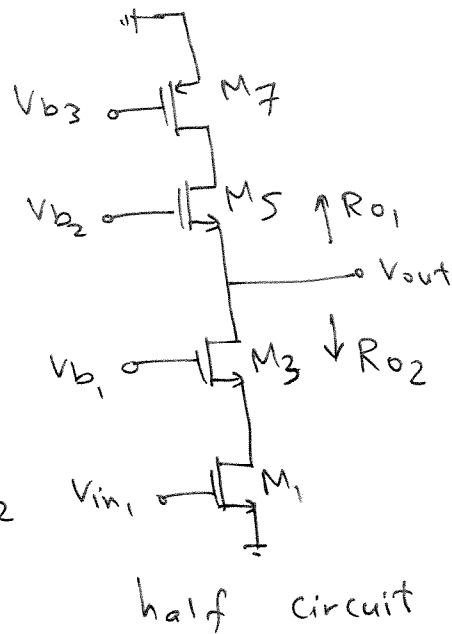
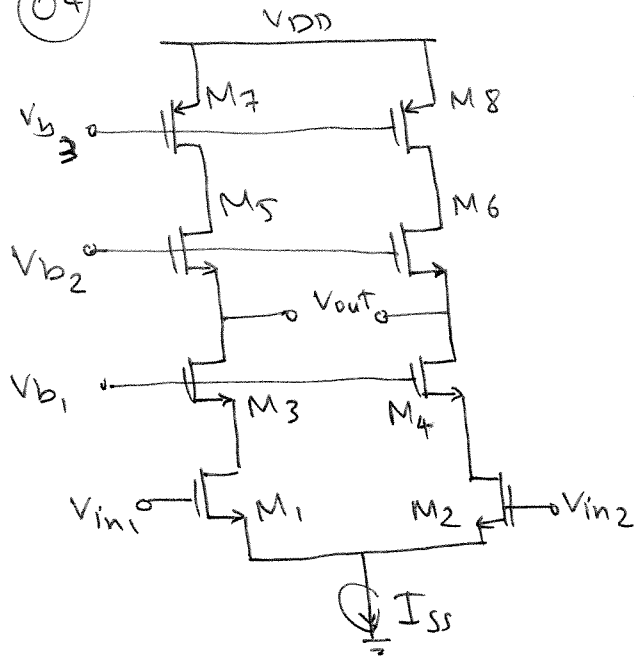
$$A_v \approx -g_{m1} \left[(g_{m3} r_{o3} r_{o1}) \parallel (g_{m5} r_{o5} r_{o7}) \right] \Rightarrow$$

$$200 = \sqrt{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{SS}} \left[\left(\sqrt{\mu_n C_{ox} \left(\frac{W}{L}\right)_3 I_{SS}} \left(\frac{2}{\lambda_n I_{SS}}\right)^2 \right) \parallel \left(\sqrt{\mu_p C_{ox} \left(\frac{W}{L}\right)_5 I_{SS}} \left(\frac{2}{\lambda_p I_{SS}}\right)^2 \right) \right]$$

$$\Rightarrow 200 = \sqrt{10^{-4} \left(\frac{W}{L}\right)_1 10^{-3}} \left[\left(\sqrt{10^{-4} \left(\frac{W}{L}\right)_3 10^{-3}} \left(\frac{20}{10^{-3}}\right)^2 \right) \parallel \left(\sqrt{0.5 \times 10^{-4} \left(\frac{W}{L}\right)_5 10^{-3}} \left(\frac{10}{10^{-3}}\right)^2 \right) \right]$$

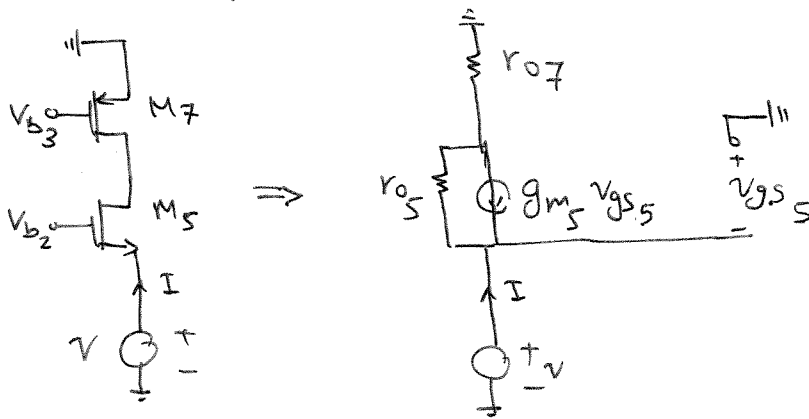
$$\Rightarrow \frac{W}{L} = 33.28$$

(64)



$$R_{o2} = g_{m3} r_{o3} r_{o1} + r_{o1} + r_{o3}$$

To calculate R_{o1} , using the small signal model we have:

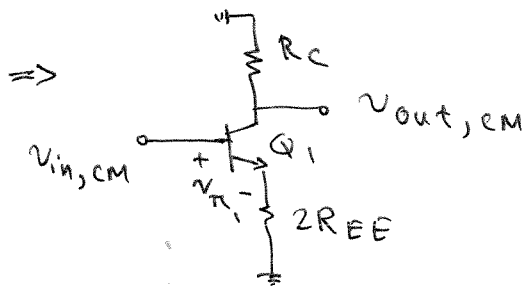
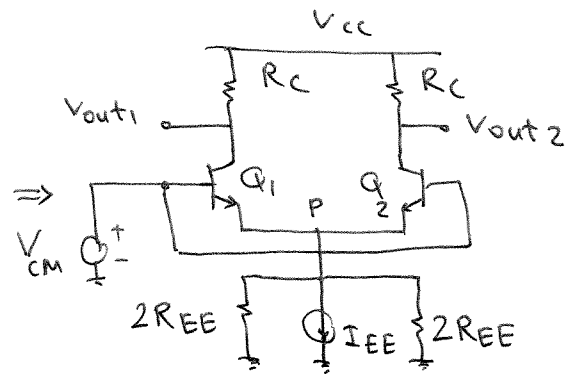
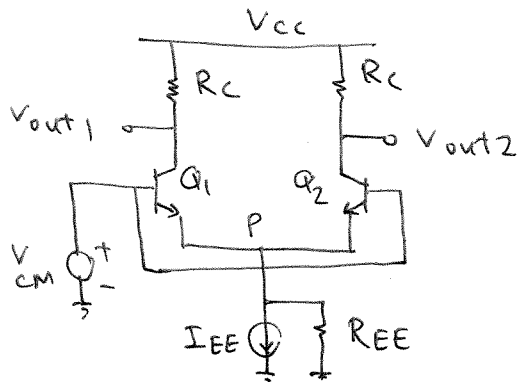


$$v_{gs5} = -v \rightarrow g_{m5} v_{gs5} = -g_{m5} v$$

From KVL: $v = r_{o5} (I - g_{m5} v) + r_{o7} I$

$$\rightarrow \frac{v}{I} = R_{o1} = \frac{r_{o5} + r_{o7}}{1 + g_{m5} r_{o5}} \Rightarrow A_v = -g_{m1} (R_{o1} \parallel R_{o2})$$

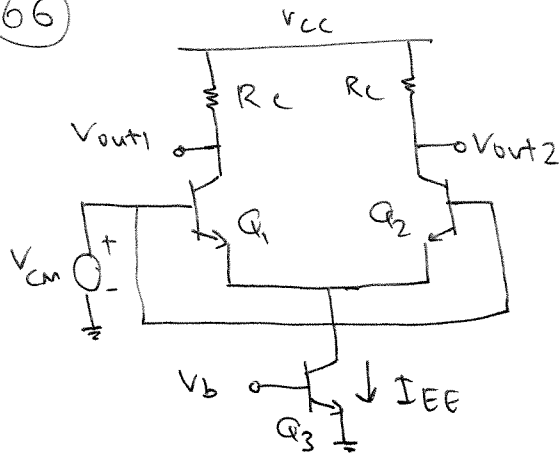
(65)



$$V_{out,CM} = -g_{m1} v_{\pi 1} R_C = -g_{m1} R_C \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + 2R_{EE}} v_{in,CM}$$

$$\Rightarrow \frac{V_{out,CM}}{v_{in,CM}} = - \frac{g_{m1} R_C}{1 + 2R_{EE} g_{m1}} = - \frac{\frac{R_C}{2}}{R_{EE} + \frac{1}{2g_{m1}}}$$

66

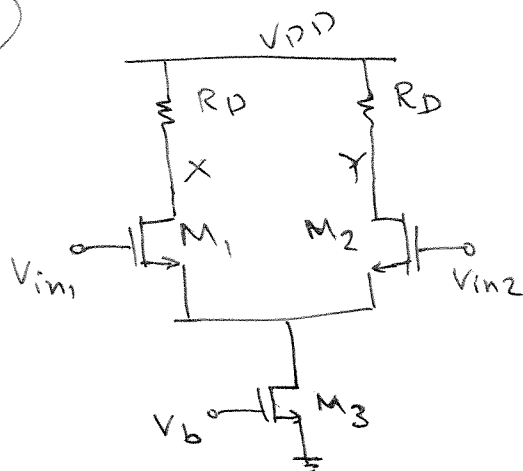


$$A_{cm} = \frac{\Delta V_{out, CM}}{\Delta V_{in, CM}} = - \frac{R_c/2}{\frac{1}{2g_m} + r_{o3}} \Rightarrow$$

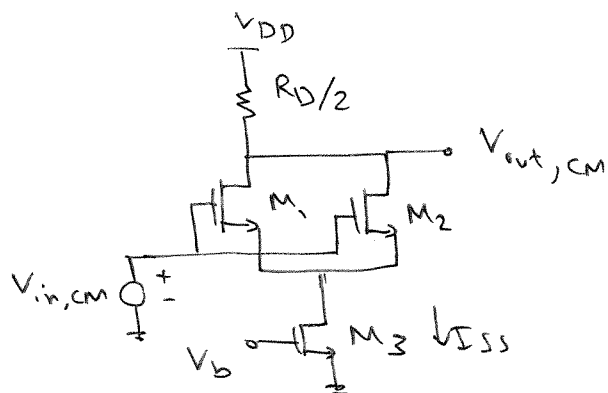
$$A_{cm} \leq 0.01 \Rightarrow \frac{R_c/2}{\frac{1}{2 \frac{I_{EE}}{2V_T}} + \frac{V_A}{I_{EE}}} < 0.01 \Rightarrow$$

$$\frac{R_c I_{EE}}{2 (V_A + V_T)} < 0.01 \Rightarrow R_c I_{EE} < 0.02 (V_A + V_T)$$

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The same value for the inputs common-mode leads to the following circuit:

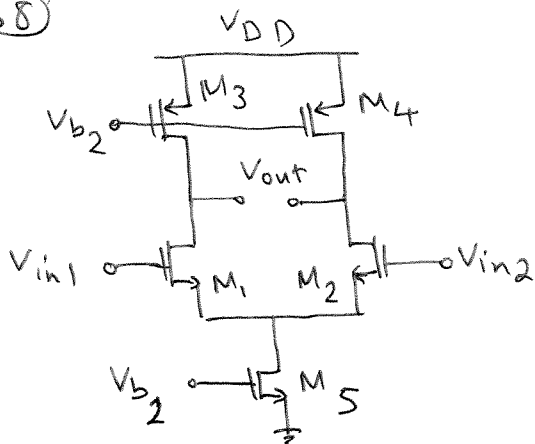


$$g_{m1} = g_{m2} = \frac{2 I_{SS}/2}{(V_{GS} - V_{TH})_{eq}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{eq}}$$

$$\frac{\Delta V_{out, CM}}{\Delta V_{in, CM}} = - \frac{R_D/2}{\frac{1}{2g_{m1}} + r_{o3}} = \frac{- R_D}{\frac{1}{g_{m1}} + 2r_{o3}} = \frac{- R_D}{\frac{(V_{GS} - V_{TH})_{eq}}{I_{SS}} + \frac{2}{\lambda I_{SS}}} \Rightarrow$$

$$A_{CM} = - \frac{R_D I_{SS}}{\frac{2}{\lambda} + (V_{GS} - V_{TH})_{eq}}$$

(68)



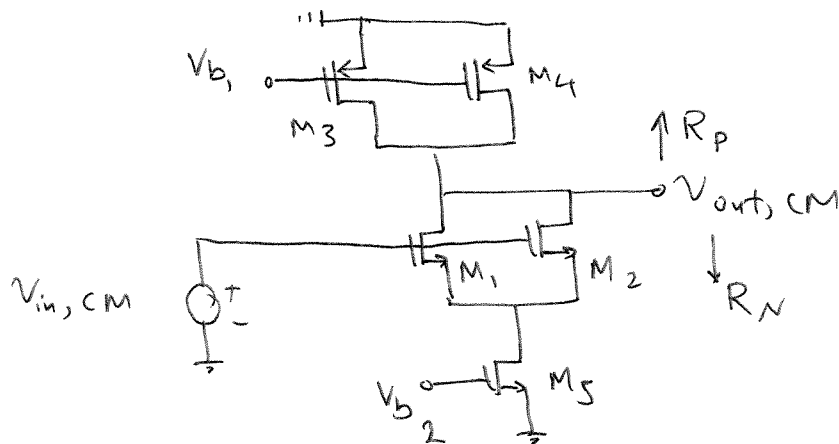
$$\lambda > 0, g_m r_o \gg 1$$

$$r_{o3} = r_{o4}$$

$$r_{o1} = r_{o2}$$

$$g_{m1} = g_{m2}$$

For the common mode input we have:



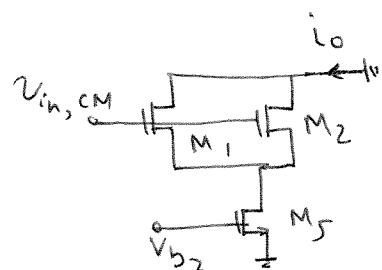
$$R_P = r_{o3} \parallel r_{o4} = \frac{r_{o3}}{2} = \frac{r_{o4}}{2}$$

$$R_N = r_{o5} + \frac{r_{o1}}{2} + 2g_{m1} \frac{r_{o1}}{2} r_{o5} =$$

$$g_{m1} r_{o1} r_{o5} + r_{o5} + \frac{r_{o1}}{2}$$

$$\frac{i_o}{v_{in,CM}} = G_m = \frac{2g_{m1} v_{gs1}}{v_{in,CM}} = 2g_{m1} \frac{\frac{1}{2g_{m1}}}{\frac{1}{2g_{m1}} + r_{o5}}$$

$$\rightarrow G_m = \frac{2g_{m1}}{1 + 2g_{m1} r_{o5}} \approx \frac{1}{r_{o5}}$$



$$\Rightarrow A_{CM} = - G_m R_o$$

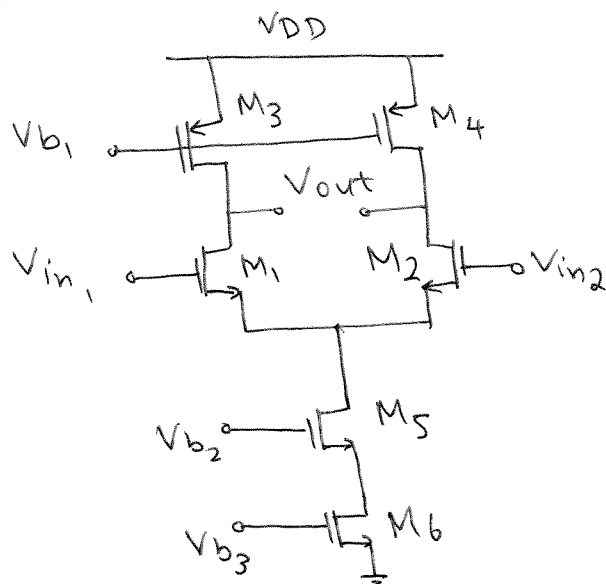
$$R_o = R_P \parallel R_N = \frac{r_{o4}}{2} \parallel (g_{m1} r_{o1} r_{o5} + r_{o5} + \frac{r_{o1}}{2})$$

$$\approx \frac{r_{o4}}{2} \parallel g_{m1} r_{o1} r_{o5} \approx \frac{r_{o4}}{2}$$

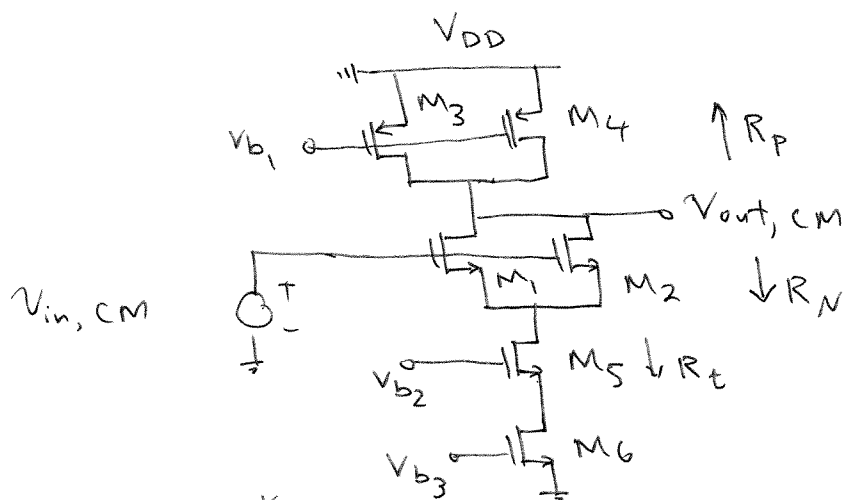
$$\rightarrow A_{CM} = - \frac{1}{r_{o5}} \frac{r_{o4}}{2} = - \frac{r_{o4}}{2 r_{o5}}$$

(69)

(a)



For the common mode input:



$$R_P = r_{o3} \parallel r_{o4} = \frac{r_{o3}}{2}$$

$$R_N = \frac{r_{o1}}{2} + R_t + 2g_{m1} \frac{r_{o1}}{2} R_t \approx g_{m1} r_{o1} R_t \approx$$

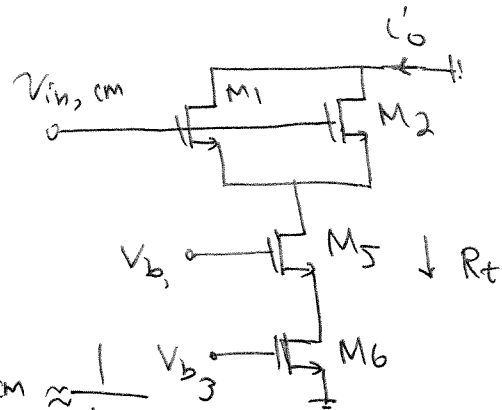
$$g_{m1} r_{o1} g_{m5} r_{o5} r_{o6}$$

$$R_{out} = R_P \parallel R_N = \frac{r_{o3}}{2} \parallel g_{m1} g_{m5} r_{o1} r_{o5} r_{o6} \approx \frac{r_{o3}}{2}$$

To calculate G_m :

$$G_m = \frac{i_o}{v_{in, CM}} = \frac{2g_{m1} v_{gs1}}{v_{in, CM}}$$

$$= \frac{2g_{m1}}{v_{in, CM}} \frac{\frac{1}{2g_{m1}}}{\frac{1}{2g_{m1}} + R_t} v_{in, CM} \approx \frac{1}{R_t} v_{in, CM}$$

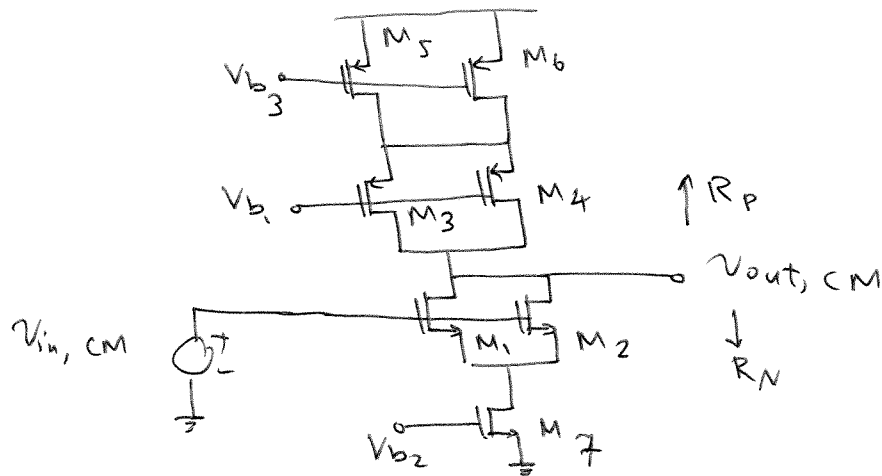


$$\rightarrow A_{CM} = -G_m R_{out} = -\frac{r_{o3}}{2 R_t} =$$

$$-\frac{r_{o3}}{2 g_{m5} r_{o5} r_{o6}}$$

(69) (b)

For the common mode input, the circuit is:

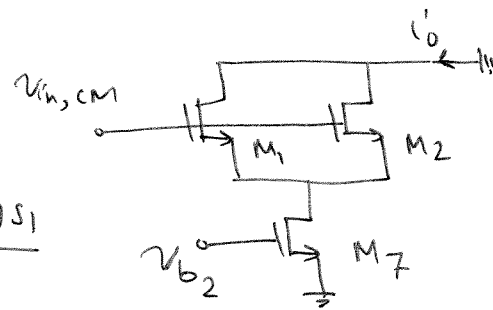


$$R_P = 2g_{m3} \frac{r_{o3}}{2} \frac{r_{o5}}{2} + \frac{r_{o3}}{2} + \frac{r_{o5}}{2} \approx \frac{g_{m3} r_{o3} r_{o5}}{2}$$

$$R_N = 2g_{m1} \frac{r_{o1}}{2} r_{o7} + \frac{r_{o1}}{2} + r_{o7} \approx g_{m1} r_{o1} r_{o7}$$

$$R_{out} = R_N \parallel R_P$$

To calculate G_m :



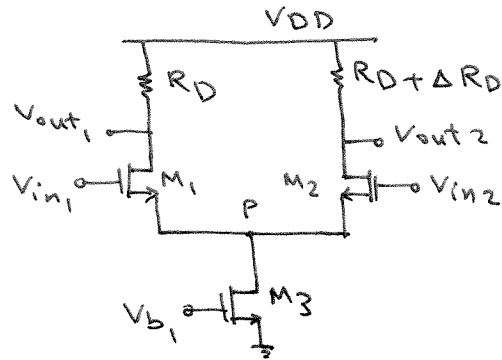
$$G_m = \frac{i_o'}{V_{in,CM}} = \frac{2g_{m1} V_{gs1}}{V_{in,CM}}$$

$$= \frac{2g_{m1}}{V_{in,CM}} \frac{\frac{1}{2g_{m1}}}{\frac{1}{2g_{m1}} + r_{o7}} V_{in,CM} \approx \frac{1}{r_{o7}}$$

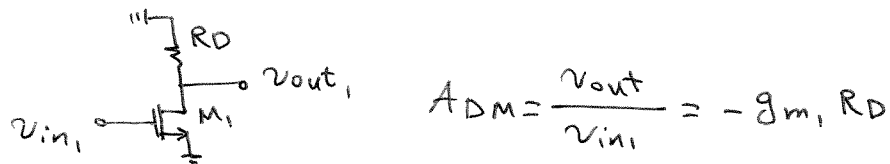
$$\rightarrow A_{cm} = -G_m R_{out}$$

(70)

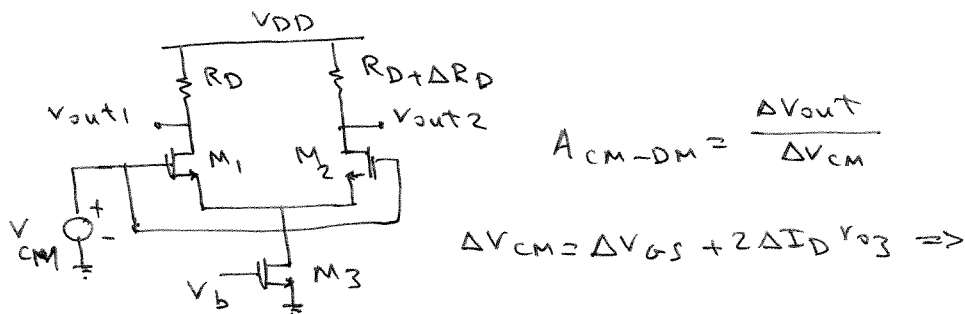
(a)



To calculate A_{DM} , using the half circuit:



To calculate, A_{CM-DM} we have:



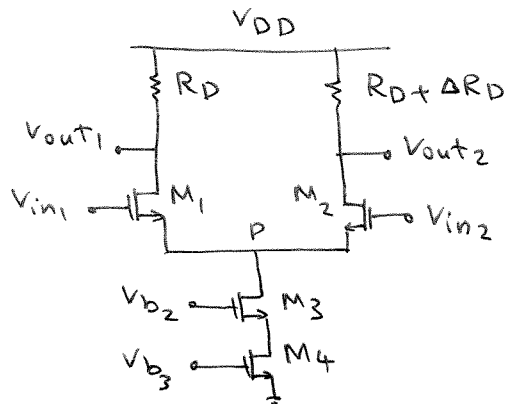
$$\Delta V_{cm} = \Delta I_D \left(\frac{1}{g_{m1}} + 2r_{o3} \right)$$

$$\Delta V_{out} = \Delta V_{out1} - \Delta V_{out2} = -\Delta R_D \Delta I_D \Rightarrow$$

$$A_{CM-DM} = - \frac{\Delta R_D}{\frac{1}{g_{m1}} + 2r_{o3}} \Rightarrow$$

$$CMRR = \frac{A_{DM}}{A_{CM-DM}} = \frac{g_{m1} R_D}{\frac{\Delta R_D}{\frac{1}{g_{m1}} + 2r_{o3}}} = (1 + 2g_{m1} r_{o3}) \frac{R_D}{\Delta R_D}$$

(70) (b)



To calculate A_{DM} , using the half circuit, we have

$$A_{DM} = \frac{V_{out1}}{V_{in1}} = -g_{m1} R_D$$

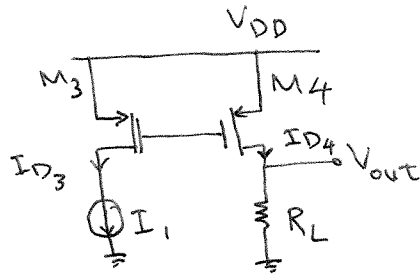
Similar to part (a) we have:

$$A_{CM-DM} = - \frac{\Delta R_D}{\frac{1}{g_{m1}} + 2 [g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}]}$$

$$\Rightarrow CMMR = \frac{A_{DM}}{A_{CM-DM}} = (1 + 2g_{m1} [g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}]) \frac{R_D}{\Delta R_D}$$

Notice that CMMR of part (b) is much higher than the one for part (a).

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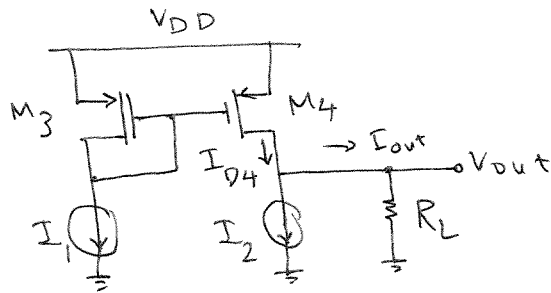
$$\left(\frac{W}{L}\right)_3 = N \left(\frac{W}{L}\right)_4$$

$$\left(\frac{W}{L}\right)_3 = N \left(\frac{W}{L}\right)_4 \Rightarrow I_{D3} = N I_{D4} \Rightarrow \underbrace{i_{d3} = N i_{d4}}_{\text{small signal}}$$

$$\begin{cases} i_{d3} = i_1 \\ v_{out} = R_L i_{d4} = \frac{R_L}{N} i_{d3} = \frac{R_L}{N} i_1 \Rightarrow \end{cases}$$

$$\frac{v_{out}}{i_1} = \frac{R_L}{N}$$

72



$$(a) \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4$$

if $I_1 = I_2 = I_0 \Rightarrow \begin{cases} V_{out} = I_{out} \times R_L = 0 & \text{because} \\ I_{out} = I_{D4} - I_2 = I_1 - I_2 = 0 \end{cases}$

$$\text{if } I_1 = I_0 + \Delta I \Rightarrow I_{D4} = I_{D3} = I_1 = I_0 + \Delta I$$

$$I_2 = I_0 - \Delta I \Rightarrow I_{out} = I_{D4} - I_2 = 2\Delta I$$

$$V_{out} = I_{out} R_L = 2 R_L \Delta I$$

$$(b) \left(\frac{W}{L}\right)_3 = 2\left(\frac{W}{L}\right)_4$$

$$\Rightarrow I_{D3} = 2 I_{D4}$$

$$\text{if } I_1 = I_2 = I_0 \text{ then } I_{D3} = I_1 = I_0 \Rightarrow I_{D4} = \frac{I_{D3}}{2}$$

$$\Rightarrow I_{D4} = \frac{I_0}{2} \Rightarrow I_{out} = I_{D4} - I_2 = -\frac{I_0}{2}$$

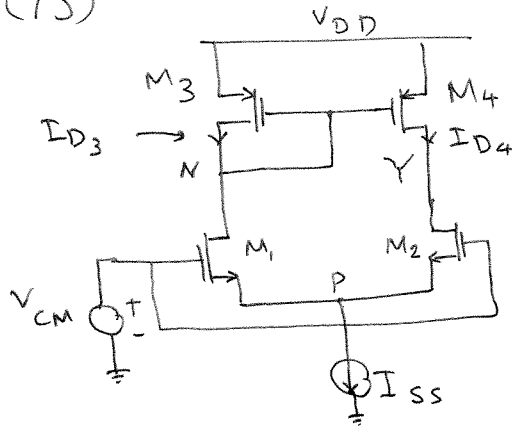
$$\Rightarrow V_{out} = R_L I_{out} = -\frac{R_L I_0}{2}$$

$$\text{if } I_1 = I_0 + \Delta I \Rightarrow I_{D4} = \frac{I_{D3}}{2} = \frac{I_1}{2} = \frac{I_0 + \Delta I}{2}$$

$$I_2 = I_0 - \Delta I \Rightarrow I_{out} = I_{D4} - I_2 = -\frac{I_0}{2} + \frac{3\Delta I}{2}$$

$$\Rightarrow V_{out} = R_L I_{out} = + R_L \left(-\frac{I_0}{2} + \frac{3\Delta I}{2}\right)$$

(73)



$$V_{TH3} = |V_{TH,P}|$$

$$(a) \quad I_{D3} = \frac{I_{SS}}{2} = \frac{1}{2} \mu_P C_{ox} \left(\frac{W}{L} \right)_3 (V_{SG3} - V_{TH})^2 \Rightarrow$$

$$V_{SG3} = V_{TH3} + \sqrt{\frac{I_{SS}}{\mu_P C_{ox} \left(\frac{W}{L} \right)_3}}$$

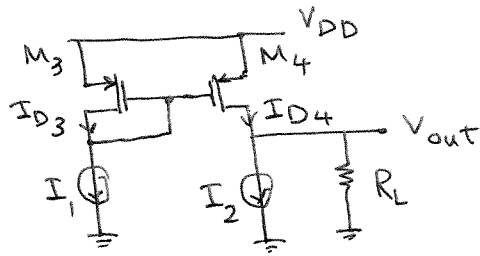
$$V_N = V_{DD} - V_{SG3} = V_{DD} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_P C_{ox} \left(\frac{W}{L} \right)_3}}$$

(b) Since the circuit is symmetric:

$$\begin{cases} I_{D3} = I_{D4} \\ V_{SG3} = V_{SG4} \end{cases} \Rightarrow V_{SD3} = V_{SD4} \Rightarrow V_Y = V_N$$

(c) if V_{DD} changes by a small amount ΔV , both V_N and V_Y will change by ΔV .

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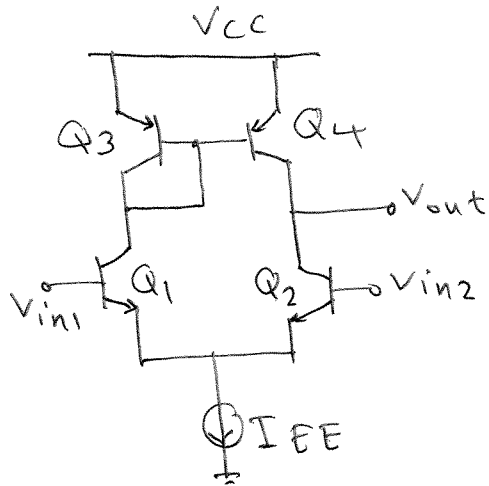


$$I_{D3} = I_{D4} = I_1$$

$$V_{out} = (I_{D4} - I_2) R_L = (I_1 - I_2) R_L$$

$$\begin{matrix} \text{small} \\ \Rightarrow \\ \text{signal} \end{matrix} \quad \frac{V_{out}}{i_1} = R_L \quad , \quad \frac{V_{out}}{i_2} = -R_L$$

(75)



$$V_{A,n} = 5V$$

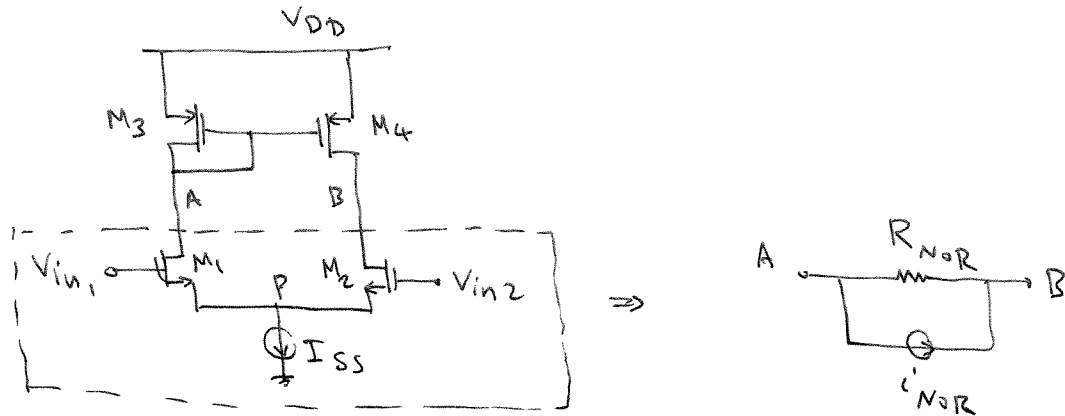
$$A_v = 100$$

$$\frac{v_{out}}{v_{in1} - v_{in2}} = g_{mN} (r_{oN1} || r_{oP}) =$$

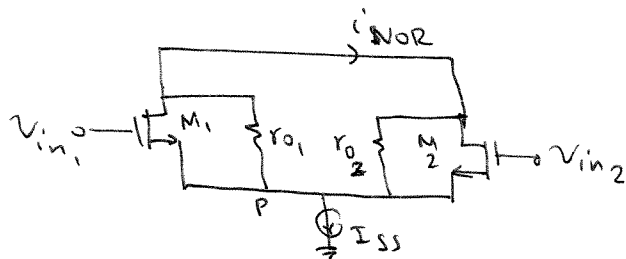
$$\frac{I_{EE}/2}{V_T} \left(\frac{V_{A,n}}{I_{EE}/2} || \frac{V_{A,p}}{I_{EE}/2} \right) = \frac{V_{A,n} V_{A,p}}{(V_{A,n} + V_{A,p}) V_T}$$

$$\Rightarrow 100 = \frac{5 V_{A,p}}{(5 + V_{A,p}) 0.026} \Rightarrow V_{A,p} = 5.417V$$

(76)



To calculate i'_{NOR} we have:

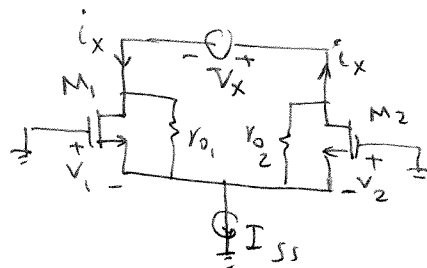


$$r_{o1} (i'_{NOR} + g_{m1} V_{in1}) + r_{o2} (i'_{NOR} - g_{m2} V_{in2}) = 0$$

$$\Rightarrow 2 r_{oN} i'_{NOR} = -g_{m1} r_{oN} (V_{in1} - V_{in2}) \Rightarrow$$

$$i'_{NOR} = -\frac{g_{mN}}{2} (V_{in1} - V_{in2})$$

To calculate R_{NOR} :

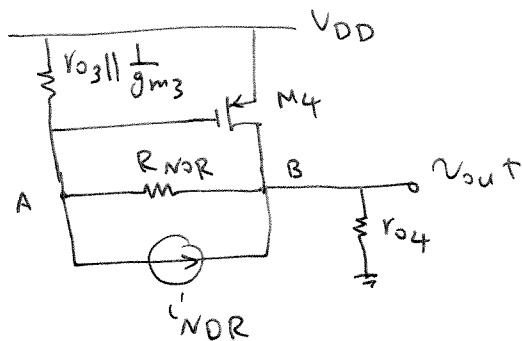


$$V_1 = V_2$$

$$(i_x - g_{m1} V_1) r_{o1} + (i_x + g_{m2} V_2) r_{o2} = V_x$$

$$\Rightarrow R_{NOR} = \frac{V_x}{i_x} = 2 r_{oN}$$

Therefore, utilizing the Norton model we have:



$$\begin{cases} \frac{V_A - V_B}{R_{NOR}} + \frac{V_A}{r_{o3} \parallel \frac{1}{g_{m3}}} + i_{NOR} = 0 \Rightarrow V_A = \frac{\frac{V_B}{R_{NOR}} - i_{NOR}}{\frac{1}{R_{NOR}} + \frac{1}{r_{o3} \parallel \frac{1}{g_{m3}}}} \\ \frac{V_B - V_A}{R_{NOR}} + \frac{V_B}{r_{o4}} - i_{NOR} + g_{m4} V_A = 0, V_B = v_{out} \end{cases}$$

$$\Rightarrow v_{out} \left(\frac{1}{R_{NOR}} + \frac{1}{r_{o4}} \right) + \left(g_{m4} - \frac{1}{R_{NOR}} \right) \frac{\frac{v_{out}}{R_{NOR}} - i_{NOR}}{\frac{1}{R_{NOR}} + \frac{1}{r_{o3} \parallel \frac{1}{g_{m3}}}} = i_{NOR}$$

$\frac{1}{g_{m3}} \ll r_{o3}, \frac{1}{g_{m3}} \ll R_{NOR}, g_{m3} = g_{m4} = g_m, r_{o3} = r_{o4} = r_{op}$

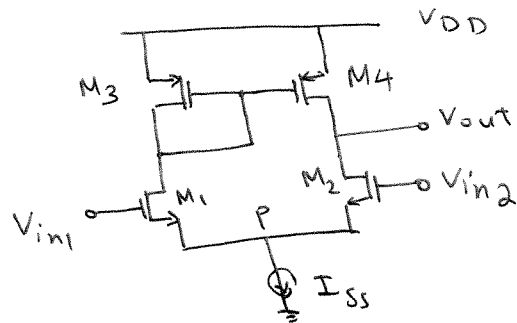
$$\Rightarrow v_{out} \left(\frac{1}{R_{NOR}} + \frac{1}{r_{op}} \right) + g_{m4} \frac{\frac{v_{out}}{R_{NOR}} - i_{NOR}}{g_{m3}} = i_{NOR}$$

$$\Rightarrow v_{out} \left(\frac{1}{R_{NOR}} + \frac{1}{r_{op}} \right) + \frac{v_{out}}{R_{NOR}} = 2 i_{NOR} \Rightarrow$$

$$\frac{2 v_{out}}{R_{NOR}} + \frac{v_{out}}{r_{op}} = 2 i_{NOR} \Rightarrow v_{out} \left(\frac{1}{r_{on}} + \frac{1}{r_{op}} \right) = -g_{mN} (v_{in1} - v_{in2})$$

$$\Rightarrow \frac{v_{out}}{v_{in1} - v_{in2}} = -g_{mN} (r_{on} \parallel r_{op})$$

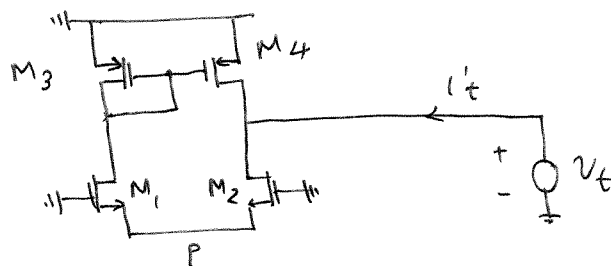
(77)



$$g_m r_o \gg 1$$

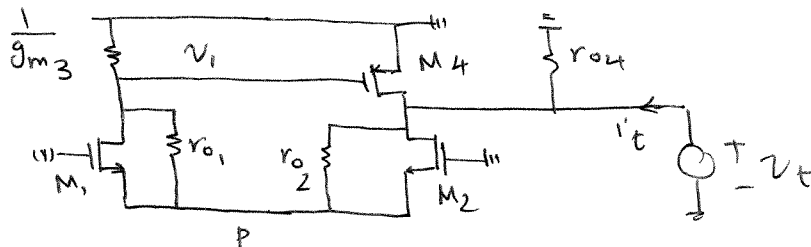
$$g_{m1} = g_{m2}$$

To calculate the output impedance we have the following circuit:



$$R_{out} = \frac{v_t}{i_t}$$

↓ neglecting r_{o3} ($r_{o3} \gg \frac{1}{g_{m3}}$)

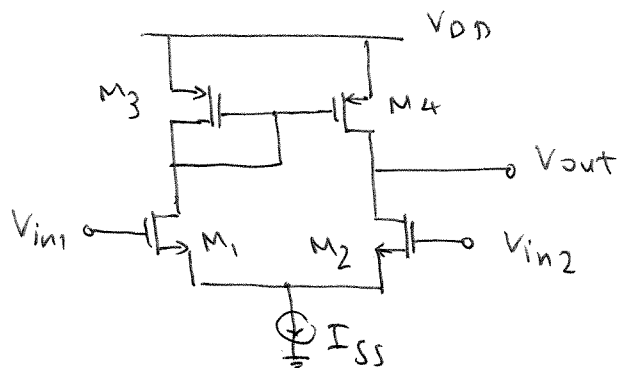


writing node equations of v_1 and v_p :

$$\begin{cases} g_{m3} v_1 + \frac{v_1 - v_p}{r_{o1}} - g_{m1} v_p = 0 \\ 2g_{m1} v_p + \frac{v_p - v_1}{r_{o1}} + \frac{v_p - v_t}{r_{o2}} = 0 \end{cases} \xRightarrow{g_m r_o \gg 1} \begin{cases} g_{m3} v_1 \approx g_{m1} v_p \\ 2g_{m1} v_p \approx 0 \end{cases}$$

$$\Rightarrow v_p \approx v_1 = 0 \Rightarrow R_{out} = \frac{v_t}{i_t} = r_{o4} \parallel r_{o2} = r_{oN} \parallel r_{oP}$$

78

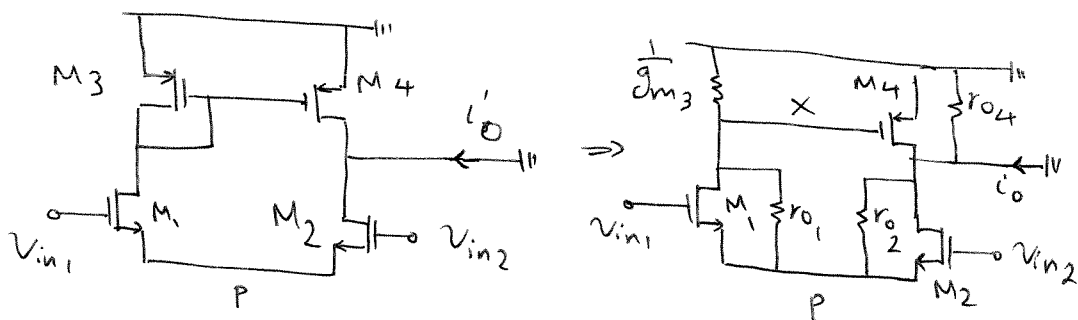


$$g_m r_o \gg 1$$

$$g_{m1} = g_{m2}$$

$$g_{m3} = g_{m4}$$

To calculate G_m we have from small signal model:



writing node equations of nodes P and X we have:

$$\begin{cases} g_{m1}(v_P - v_{in1}) + g_{m2}(v_P - v_{in2}) + \frac{v_P - v_X}{r_{o1}} + \frac{v_P}{r_{o2}} = 0 \\ g_{m3} v_X + g_{m1}(v_{in1} - v_P) + \frac{v_X - v_P}{r_{o1}} = 0 \end{cases}$$

Since $g_m r_o \gg 1$ we have

$$\begin{cases} g_{m1}(v_P - v_{in1}) + g_{m2}(v_P - v_{in2}) = 0 \Rightarrow v_P = \frac{v_{in1} + v_{in2}}{2} \\ v_X = -\frac{g_{m1}}{g_{m3}}(v_{in1} - v_P) = -\frac{g_{m1}}{g_{m3}}\left(\frac{v_{in1} - v_{in2}}{2}\right) \end{cases}$$

$$i_o = -\frac{v_P}{r_{o2}} + g_{m2}(v_{in2} - v_P) - g_{m4}(-v_X)$$

$$\Rightarrow i_o \approx -g_{m4}(-v_x) + g_{m2} \left(v_{in2} - \frac{v_{in1} + v_{in2}}{2} \right)$$

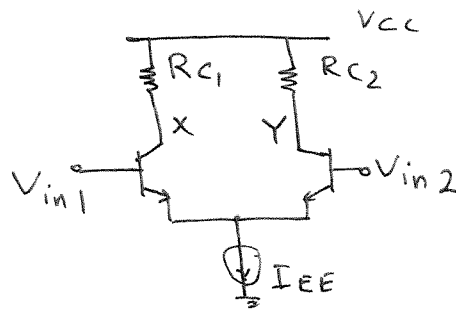
$$= - \left[g_{m4} \frac{g_{m1}}{g_{m3}} \left(\frac{v_{in1} - v_{in2}}{2} \right) + g_{m2} \left(\frac{v_{in1} - v_{in2}}{2} \right) \right]$$

$$= -g_{m1} (v_{in1} - v_{in2})$$

$$G_m = \frac{i_o}{v_{in1} - v_{in2}} = -g_{m1} = -g_{mN}$$

$$\rightarrow A_v = -G_m R_{out} = g_{mN} (r_{oN} \parallel r_{op})$$

79



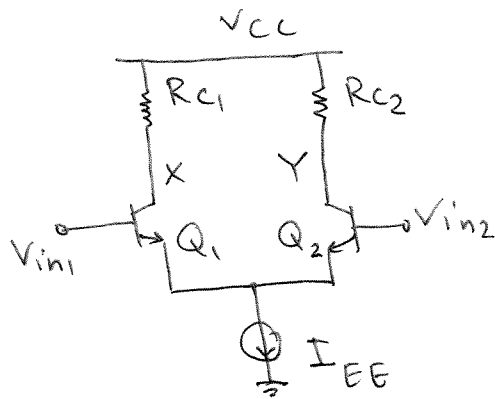
$$\begin{aligned} A_v &= 10, \\ P &= 2 \text{ mW} \\ V_{CC} &= 2.5 \text{ V} \\ V_A &= \infty \end{aligned}$$

$$P = V_{CC} I_{EE} \Rightarrow 2 \times 10^{-3} = 2.5 I_{EE} \Rightarrow I_{EE} = 0.8 \text{ mA}$$

$$A_v = \frac{v_{XY}}{v_{in1} - v_{in2}} = -g_m R_C = -\frac{I_{EE}/2}{V_T} R_C$$

$$\Rightarrow 10 = \frac{0.4 \times 10^{-3}}{0.026} R_C \Rightarrow R_C = 650 \Omega$$

(80)



$$V_{in, CM} = 1.2 \text{ V}$$

$$P = 3 \text{ mW}$$

$$V_{CC} = 2.5 \text{ V}$$

$$P = I_{EE} V_{CC} \Rightarrow 3 \times 10^{-3} = 2.5 I_{EE} \Rightarrow I_{EE} = 1.2 \text{ mA}$$

$$A_v = -g_{m1} R_C = -\frac{I_{EE}/2}{V_T} R_C = -\frac{R_C I_{EE}}{2 V_T}$$

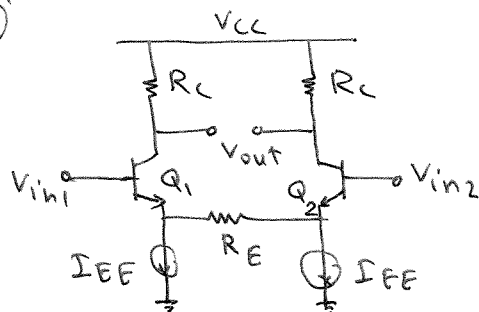
To maximize gain, $R_C I_{EE}$ and therefore R_C should be maximum. However, the upper bound of R_C value is limited by the voltage value of X . because:

$$V_{in, CM} \leq V_X \Rightarrow 1.2 \leq V_{CC} - R_C I_{EE}/2 \Rightarrow$$

$$R_C \leq 2 \frac{V_{CC} - 1.2}{I_{EE}} \Rightarrow R_C \leq 2 \frac{2.5 - 1.2}{1.2 \times 10^{-3}} \Rightarrow$$

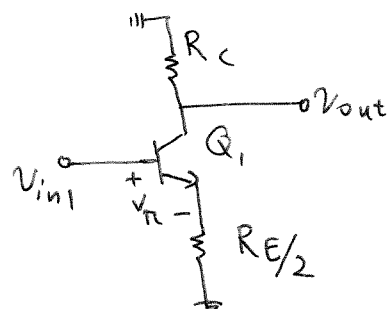
$$R_C \leq 2.167 \text{ k}\Omega \Rightarrow R_C = 2.167 \text{ k}\Omega$$

81)



$$\begin{aligned} A_v &= 5 \\ P &= 4 \text{ mW} \\ V_{CC} &= 2.5 \text{ V} \\ V_A &= \infty \end{aligned}$$

The half circuit is:



$$\begin{aligned} A_v &= \frac{v_{out}}{v_{in1}} \approx \frac{-g_{m1} v_{\pi} R_C}{v_{in1}} = \\ &= - \frac{g_{m1} R_C}{v_{in1}} \frac{1}{\frac{1}{g_{m1}} + \frac{R_E}{2}} v_{in1} \\ &= - \frac{R_C}{\frac{R_E}{2} + \frac{1}{g_{m1}}} \end{aligned}$$

$$P = 4 \text{ mW} = 2 I_{EE} V_{CC} = 5 I_{EE} \Rightarrow I_{EE} = 0.8 \text{ mA}$$

$$g_m = \frac{I_{EE}}{V_T} = 0.03077$$

$$A_v = 5 \Rightarrow \frac{R_C}{\frac{R_E}{2} + 32.5} = 5 \quad (1)$$

if I_{EE} increases by 10%, the gain will be:

$$A_v = \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} \Rightarrow 5 < \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} < 5 \times 1.02 \quad (2)$$

if I_{EE} decreases by 10%, then:

$$5 \times 0.98 < \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{0.9}} < 5 \quad (3)$$

The worse case is:

$$\left\{ \begin{array}{l} \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} = 5 \times 1.02 \quad (4) \\ \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{0.9}} = 5 \times 0.98 \quad (5) \end{array} \right.$$

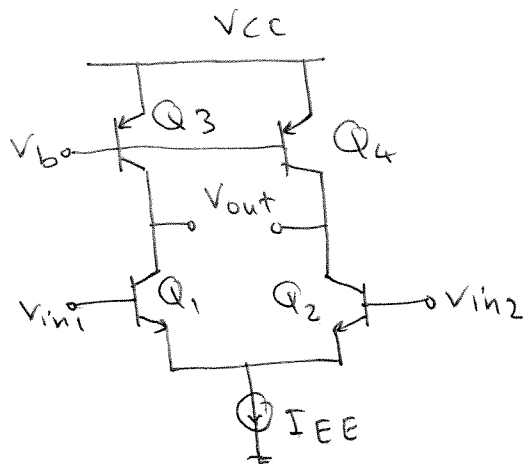
dividing (4) and (5) to (1) leads to,

$$\left\{ \begin{array}{l} \frac{\frac{R_E}{2} + 32.5}{\frac{R_E}{2} + \frac{32.5}{1.1}} = 1.02 \Rightarrow R_E = 236.36 \Omega \\ \frac{\frac{R_E}{2} + 32.5}{\frac{R_E}{2} + \frac{32.5}{0.9}} = 0.98 \Rightarrow R_E = 288.89 \Omega \end{array} \right.$$

To ensure less than 2% gain variation for 10% current variation $R_E = 288.89 \Omega$

$$\text{From (1)} \quad R_C = 5 \left(\frac{R_E}{2} + 32.5 \right) = 884.72 \Omega$$

(82)



$$A_v = 100$$

$$P = 1 \text{ mW}$$

$$V_{A,n} = 6$$

$$V_{CC} = 2.5 \text{ V}$$

$$P = 1 \text{ mW} = I_{EE} V_{CC} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4 \text{ mA}$$

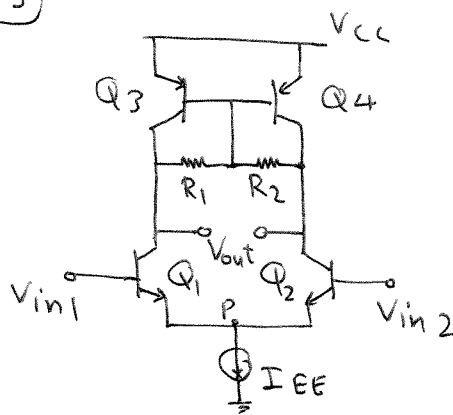
$$r_{oN} = \frac{V_{A,n}}{I_{EE}/2} = \frac{6}{0.2 \times 10^{-3}} = 30 \text{ k}\Omega, \quad g_{mN} = \frac{I_{EE}/2}{V_T} = \frac{0.2}{26} \text{ S}$$

$$A_v = -g_{mN} (r_{oN} \parallel r_{op}) \Rightarrow$$

$$100 = \frac{0.2}{26} (30 \times 10^3 \parallel r_{op}) \Rightarrow r_{op} = 22.94 \text{ k}\Omega$$

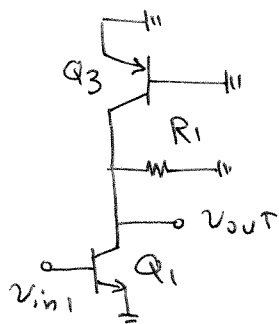
$$\Rightarrow V_{A,p} = r_{op} \frac{I_{EE}}{2} = 4.588 \text{ V}$$

83



$$\begin{aligned} A_v &= 50 \\ P &= 1 \text{ mW} \\ V_{A,n} &= 10 \text{ V} \\ V_{A,p} &= 5 \text{ V} \\ V_{CC} &= 2.5 \text{ V} \\ R_1 &= R_2 \end{aligned}$$

The half circuit is:



$$A_v = -g_{m1} (R_1 \parallel r_{o1} \parallel r_{o3})$$

$$g_{m1} = \frac{I_{EE}}{2V_T} \quad r_{o1} = \frac{2V_{A,n}}{I_{EE}} \quad r_{o3} = \frac{2V_{A,p}}{I_{EE}}$$

$$\Rightarrow 50 = \frac{1}{2V_T} ((R_1 I_{EE}) \parallel \underbrace{(2V_{A,n})}_{20} \parallel \underbrace{(2V_{A,p})}_{10}) \Rightarrow$$

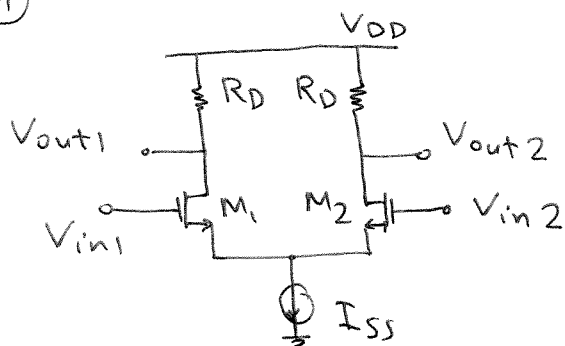
$$R_1 I_{EE} = 4.262$$

$$P = 1 \text{ mW} = I_{EE} V_{CC} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4 \text{ mA}$$

$$\rightarrow R_1 = \frac{4.262}{I_{EE}} = 10.655 \text{ K}\Omega$$

Notice that larger value of R_1 , requires smaller I_{EE} and saves power.

84



$$\Delta V_{in, \max} = 0.3 \text{ V}$$

$$P = 3 \text{ mW}$$

$$R_D = 500 \Omega$$

$$\lambda = 0, \mu_n C_{ox} = 100 \mu\text{A}/\text{V}^2$$

$$V_{DD} = 1.8 \text{ V}$$

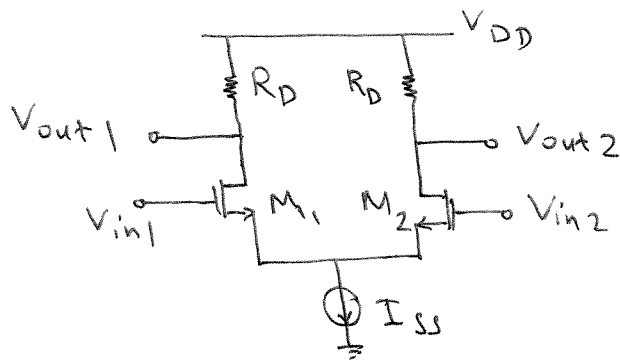
$$P = V_{DD} I_{SS} \Rightarrow 3 \times 10^{-3} = 1.8 I_{SS} \Rightarrow$$

$$I_{SS} = 1.67 \text{ mA}$$

$$\Delta V_{in, \max} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$0.3 = \sqrt{\frac{2 \times 1.67 \times 10^{-3}}{10^{-4} \times \frac{W}{L}}} \Rightarrow \frac{W}{L} = 370.37$$

(85)



$$\begin{aligned} P &= 2 \text{ mW} \\ \text{overdrive} &= 100 \text{ mV} \\ V_{CM} &= 1 \text{ V} \\ \lambda &= 0, \mu_n C_{ox} = 100 \mu\text{A/V}^2 \\ V_{DD} &= 1.8 \text{ V} \\ V_{TH,n} &= 0.5 \end{aligned}$$

$$P = I_{SS} V_{DD} \Rightarrow 2 \times 10^{-3} = 1.8 I_{SS} \Rightarrow I_{SS} = 1.11 \text{ mA}$$

$$V_{GS1} - V_{TH} = \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$0.1^2 = \frac{1.11 \times 10^{-3}}{10^{-4} \times \frac{W}{L}} \Rightarrow \frac{W}{L} = 1111.11$$

$$g_m = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} = \sqrt{10^{-4} \times 1111.11 \times 1.11 \times 10^{-3}} = 0.011$$

To place the transistor at the edge of triode region:

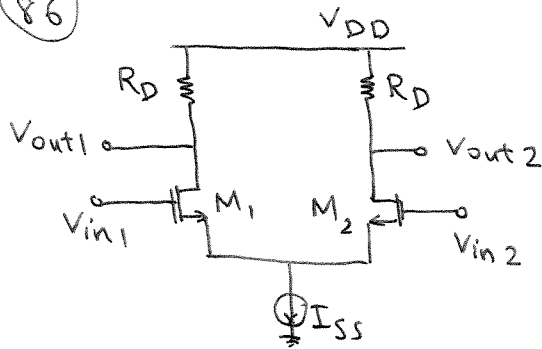
$$V_{in,CM} = V_{out1} + V_{TH,n}$$

$$1 = V_{DD} - R_D \frac{I_{SS}}{2} + 0.5 \Rightarrow$$

$$1 = 1.8 - R_D \frac{1.11 \times 10^{-3}}{2} + 0.5 \Rightarrow R_D = 2.34 \text{ k}\Omega$$

$$A_V = -g_m R_D = -25.74$$

86



$$A_v = 5$$

$$P = 1 \text{ mW}$$

$$(V_{GS} - V_{TH})_{\text{equil}} = 150 \text{ mV}$$

$$\lambda = 0, \mu_n C_{ox} = 100 \mu\text{A/V}^2$$

$$V_{DD} = 1.8 \text{ V}$$

$$P = 1 \text{ mW} = V_{DD} I_{SS} = 1.8 I_{SS} \Rightarrow I_{SS} = 0.556 \text{ mA}$$

$$g_{m1} = \frac{2 I_{D1}}{(V_{GS} - V_{TH})_{\text{equil}}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{\text{equil}}} = \frac{0.556 \times 10^{-3}}{0.15}$$

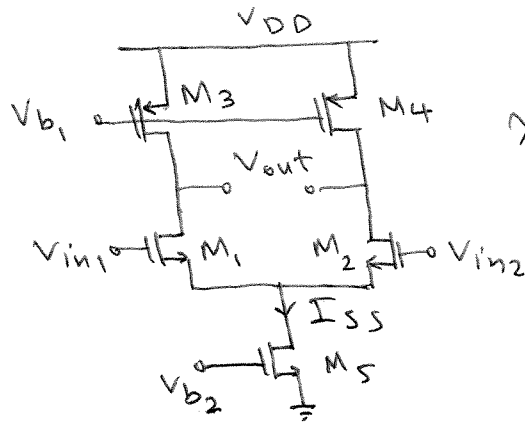
$$= 3.704 \text{ mS}$$

$$A_v = -g_{m1} R_D \Rightarrow 5 = 3.704 \times 10^{-3} \times R_D \Rightarrow R_D = 1.35 \text{ k}\Omega$$

$$(V_{GS} - V_{TH})_{\text{equil}} = \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$0.15 = \sqrt{\frac{0.556 \times 10^{-3}}{10^{-4} \times \frac{W}{L}}} \Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 246.91$$

(87)



$$A_v = 40$$

$$(V_{GS} - V_{TH})_{\text{equil}} = ?$$

$$\lambda_n = 0.1 \text{ V}^{-1} \quad \lambda_p = 0.2 \text{ V}^{-1}$$

$$\mu_n C_{ox} = 100 \mu\text{A/V}^2$$

$$\mu_p C_{ox} = 50 \mu\text{A/V}^2$$

$$V_{DD} = 1.8$$

$$P = 2 \text{ mW}$$

$$A_v = -g_{m_N} (r_{o_p} \parallel r_{o_N}) = -\frac{I_{SS}}{(V_{GS_1} - V_{TH})_{\text{equil}}} \left(\frac{1}{\frac{I_{SS}}{2} \lambda_n} \parallel \frac{1}{\frac{I_{SS}}{2} \lambda_p} \right)$$

$$= -\frac{2}{(V_{GS_1} - V_{TH})_{\text{equil}}} \left(\frac{1}{\lambda_n} \parallel \frac{1}{\lambda_p} \right) \Rightarrow$$

$$\frac{2}{(V_{GS_1} - V_{TH})_{\text{equil}}} (10 \parallel 5) = 40 \Rightarrow (V_{GS_1} - V_{TH})_{\text{equil}} = 166.67 \text{ mV}$$

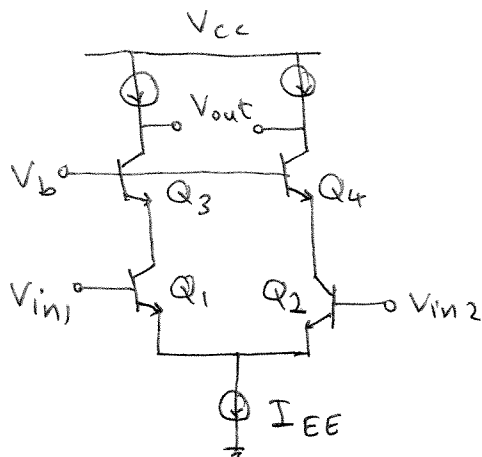
$$P = 2 \times 10^{-3} = V_{DD} I_{SS} \Rightarrow I_{SS} = \frac{2 \times 10^{-3}}{1.8} = 1.11 \text{ mA}$$

$$\left(\frac{W}{L} \right)_{1,2} = \frac{I_{SS}}{\mu_n C_{ox} (V_{GS_1} - V_{TH})_{\text{equil}}^2} = \frac{1.11 \times 10^{-3}}{10^{-4} \times (0.16667)^2} = 400$$

$$\left(\frac{W}{L} \right)_{3,4} = \frac{I_{SS}}{\mu_p C_{ox} (V_{GS_1} - V_{TH})_{\text{equil}}^2} = \frac{1.11 \times 10^{-3}}{0.5 \times 10^{-4} \times (0.16667)^2} = 800$$

$$\left(\frac{W}{L} \right)_5 = \frac{2 I_{SS}}{\mu_n C_{ox} (V_{GS_1} - V_{TH})_{\text{equil}}^2} = \frac{2 \times 1.11 \times 10^{-3}}{10^{-4} \times (0.16667)^2} = 800$$

(88)



$$A_v = 4000$$

$$\beta = 100$$

$$V_{CC} = 2.5 \text{ V}$$

$$P = 1 \text{ mW}$$

$$P = I_{EE} V_{CC} = 10^{-3} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4 \text{ mA}$$

$$g_{m1-4} = \frac{I_{EE}}{2V_T} = \frac{0.2}{26} = 7.692 \text{ mS}$$

$$r_{\pi1-4} = \frac{\beta}{g_{m1}} = 13 \text{ k}\Omega$$

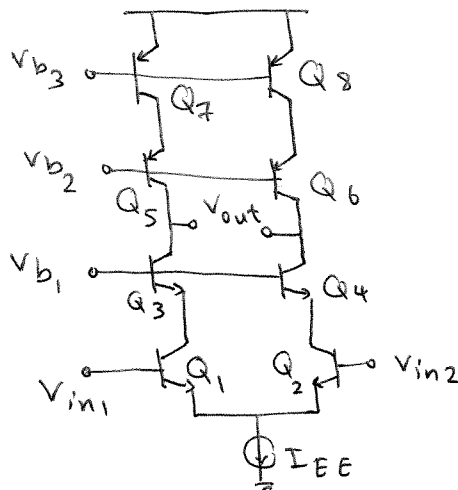
$$r_{o1-4} = \frac{V_A}{\frac{I_{EE}}{2}} = 5 \times 10^3 \text{ V}_A$$

$$A_v = -g_{m1} [g_{m3} (r_{o1} \parallel r_{\pi3}) r_{o3} + (r_{o1} \parallel r_{\pi3}) + r_{o3}] \Rightarrow$$

$$4000 = \frac{0.2}{26} \left[\frac{0.2}{26} (5 \times 10^3 \text{ V}_A \parallel 13 \times 10^3) 5 \times 10^3 \text{ V}_A + (5 \times 10^3 \text{ V}_A) \parallel 13 \times 10^3 + 5 \times 10^3 \text{ V}_A \right]$$

$$\Rightarrow V_A = 2.197$$

(89)



$$A_v = 2000$$

$$\beta_n = 100$$

$$\beta_p = 50$$

$$V_{A,n} = 5V$$

$$V_{CC} = 2.5V$$

$$P = 2mW$$

$$P = I_{EE} V_{CC} = 2 \times 10^{-3} \Rightarrow I_{EE} = \frac{2 \times 10^{-3}}{2.5} = 0.8mA$$

$$g_{m_{1-8}} = \frac{I_{EE}}{2V_T} = \frac{0.4}{26} = 0.0154 \Rightarrow$$

$$r_{\pi_{1-4}} = \frac{\beta_n}{g_{m_1}} = 6.5k\Omega \quad r_{\pi_{5-8}} = \frac{\beta_p}{g_{m_5}} = 3.25k\Omega$$

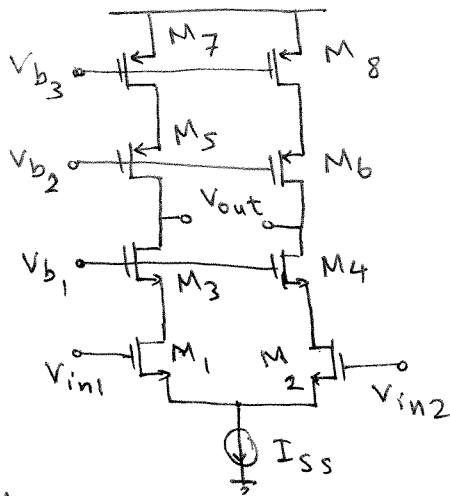
$$r_{o_{1-4}} = \frac{V_{A,n}}{I_{EE}/2} = 12.5k\Omega \quad r_{o_{5-8}} = \frac{V_{A,p}}{I_{EE}/2}$$

$$A_v \approx -g_{m_1} [g_{m_3} r_{o_3} (r_{o_1} \parallel r_{\pi_3})] \parallel [g_{m_5} r_{o_5} (r_{o_7} \parallel r_{\pi_5})]$$

$$\Rightarrow \frac{0.4}{26} \left[\frac{0.4}{26} \times 12.5 \times 10^3 (12.5 \times 10^3 \parallel 6.5 \times 10^3) \right] \parallel \left[\frac{0.4}{26} \frac{V_{A,p}}{I_{EE}/2} \left(\frac{V_{A,p}}{I_{EE}/2} \parallel 3250 \right) \right] = 2000$$

$$\Rightarrow V_{A,p} = 2.027V$$

(90)



$$A_v = 600$$

$$P = 4 \text{ mW}$$

$$(V_{GS} - V_{TH})_{NMOS} = 100 \text{ mV}$$

$$(V_{GS} - V_{TH})_{PMOS} = 150 \text{ mV}$$

$$\mu_n C_{ox} = 100 \text{ } \mu\text{A/V}^2$$

$$\mu_p C_{ox} = 50 \text{ } \mu\text{A/V}^2$$

$$\lambda_n = 0.1 \text{ V}^{-1}$$

$$A_v \approx -g_{m1} [(g_{m3} r_{o3} r_{o1}) \parallel (g_{m5} r_{o5} r_{o7})] = -600$$

$$P = 4 \text{ mW} = I_{SS} V_{DD} \Rightarrow I_{SS} = \frac{4 \times 10^{-3}}{1.8} = 2.22 \text{ mA}$$

$$g_{m1-4} = \frac{2I_{D1}}{(V_{GS} - V_{TH})_{NMOS}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{NMOS}} = \frac{2.22 \times 10^{-3}}{0.1} = 22.22 \text{ mS}$$

$$g_{m5-8} = \frac{2I_{D5}}{(V_{GS} - V_{TH})_{PMOS}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{PMOS}} = \frac{2.22 \times 10^{-3}}{0.15} = 14.815 \text{ mS}$$

$$r_{o1-4} = \frac{1}{\lambda_n \frac{I_{SS}}{2}} = \frac{1}{0.1 \times \frac{2.22}{2} \times 10^{-3}} = 9 \text{ k}\Omega$$

$$r_{o5-8} = \frac{1}{\lambda_p \frac{I_{SS}}{2}} = \frac{1}{\lambda_p \times \frac{2.22 \times 10^{-3}}{2}} = \frac{0.9 \times 10^3}{\lambda_p}$$

in A_v
 $\Rightarrow$ equation

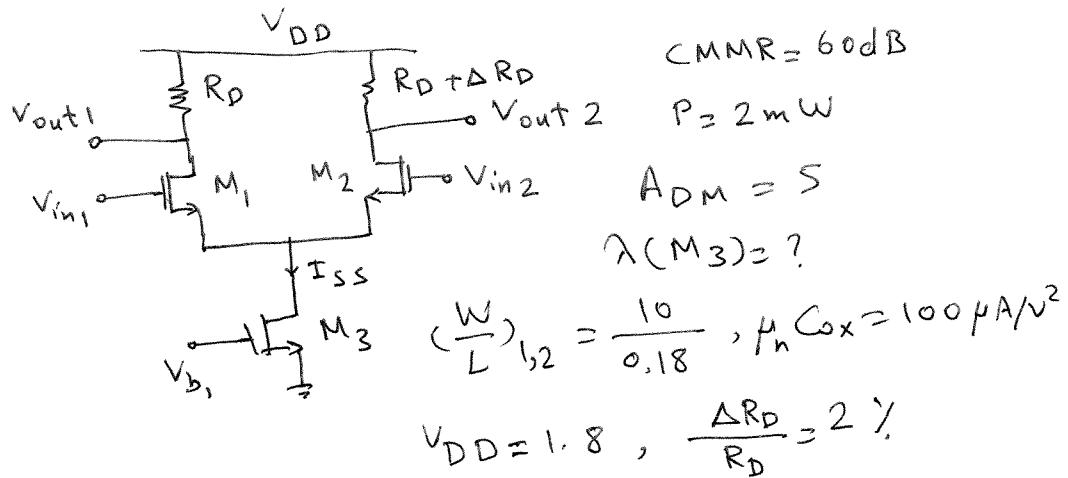
$$22.22 \times 10^{-3} \left[(22.22 \times 10^{-3} \times 81 \times 10^6) \parallel (14.815 \times 10^{-3} \times \frac{0.81 \times 10^6}{\lambda_p^2}) \right] = 600 \Rightarrow$$

$$\lambda_p = 0.66 \text{ V}^{-1}$$

$$\left(\frac{W}{L}\right)_{NMOS} = I_{SS} / (\mu_n C_{ox} (V_{GS} - V_{TH})_{NMOS}^2) = 2222.2$$

$$\left(\frac{W}{L}\right)_{PMOS} = I_{SS} / (\mu_p C_{ox} (V_{GS} - V_{TH})_{PMOS}^2) = 1975.31$$

(91)



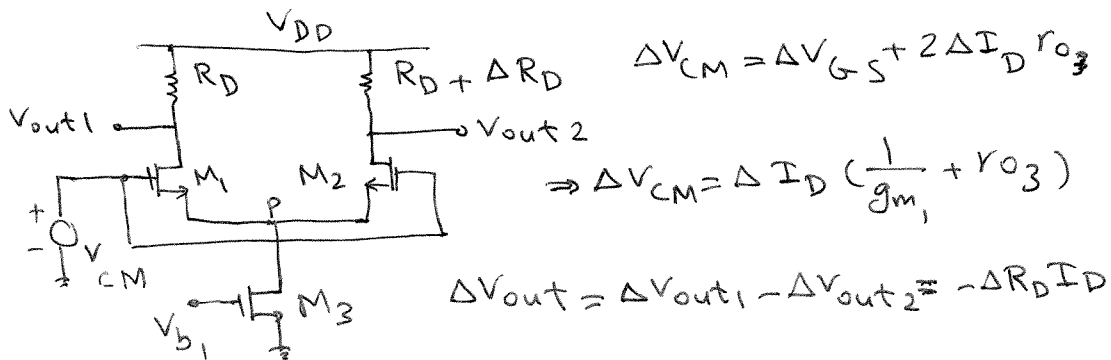
$$P = 2mW = I_{SS} V_{DD} \Rightarrow I_{SS} = \frac{2 \times 10^{-3}}{1.8} = 1.11 mA$$

$$A_{DM} = -g_{m1} R_D$$

$$g_{m1} = \sqrt{\mu_n C_{ox} (\frac{W}{L})_1 I_{SS}} = \sqrt{10^{-4} \times \frac{10}{0.18} \times 1.11 \times 10^{-3}} = 2.4845 mS$$

$$\Rightarrow R_D = \frac{|A_{DM}|}{g_{m1}} = \frac{5}{2.4845 \times 10^{-3}} = 2.012 k\Omega$$

To calculate $A_{CM,DM}$ we have:



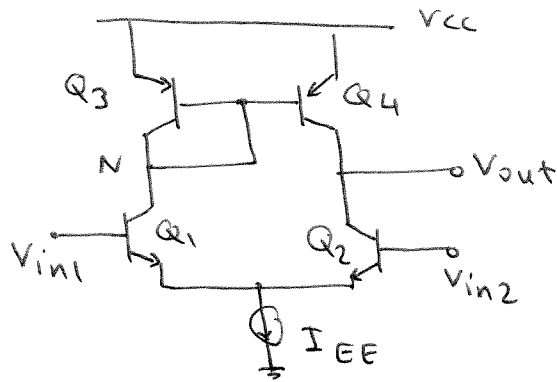
$$\Rightarrow A_{CM,DM} = \frac{\Delta V_{out}}{\Delta V_{CM}} = - \frac{\Delta R_D / 2}{\frac{1}{2g_{m1}} + r_{O3}}$$

$$\Rightarrow CMMR = \frac{A_{DM}}{A_{CM,DM}} = (1 + 2g_{m1} r_{O3}) \frac{R_D}{\Delta R_D}, r_{O3} = \frac{1}{\lambda_3 I_{SS}}$$

$$\Rightarrow \text{CMRR} = 60\text{dB} = 10^3 = \left(1 + 2 \times 2.4845 \times 10^{-3} \frac{1}{\lambda_3 \times 1.11 \times 10^{-3}}\right) 50$$

$$\Rightarrow \lambda_3 = 0.2354$$

92



$$\begin{aligned} A_v &= 200 \\ P &\approx 3 \text{ mW} \\ V_{cc} &= 2.5 \text{ V} \\ V_{A,n} &= 2 V_{A,p} \end{aligned}$$

$$P = V_{cc} I_{EE} \Rightarrow I_{EE} = \frac{3 \times 10^{-3}}{2.5} = 1.2 \text{ mA}$$

$$g_{m_{1-4}} = \frac{I_{EE}}{2V_T} = \frac{0.6}{26} = 23.077 \text{ mS}$$

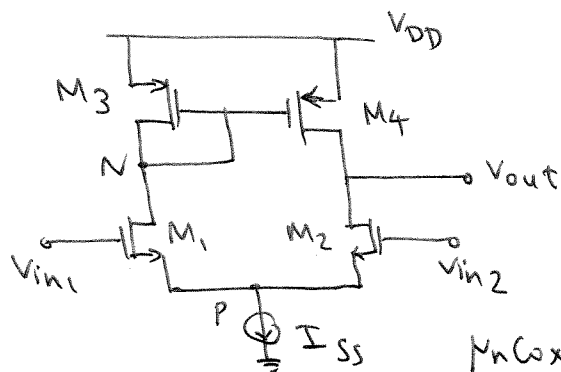
$$r_{o1} = r_{o2} = \frac{V_{A,n}}{\frac{I_{EE}}{2}} = \frac{V_{A,n}}{0.6 \times 10^{-3}}$$

$$r_{o3} = r_{o4} = \frac{V_{A,p}}{\frac{I_{EE}}{2}} = \frac{V_{A,n}}{1.2 \times 10^{-3}}$$

$$A_v = g_{m_1} (r_{o2} \parallel r_{o4}) \Rightarrow \frac{0.6}{26} \left(\frac{V_{A,n}}{0.6 \times 10^{-3}} \parallel \frac{V_{A,n}}{1.2 \times 10^{-3}} \right) = 200$$

$$\Rightarrow V_{A,n} = 15.6 \text{ V}$$

93



$$A_v = 20$$

$$P = 1 \text{ mW}$$

$$V_{DD} = 1.8 \text{ V}$$

$$V_{in, \text{cm}} = 1 \text{ V}$$

$$\mu_n C_{ox} = 2 \mu_p C_{ox} = 100 \mu\text{A/V}^2$$

$$V_{TH, n} = 0.5 \text{ V}, V_{TH, p} = -0.4 \text{ V}$$

$$\lambda_n = \frac{\lambda_p}{2} = 0.1 \text{ V}^{-1}$$

$$P = V_{DD} I_{SS} \Rightarrow I_{SS} = \frac{10^{-3}}{1.8} = 0.556 \text{ mA}$$

$$A_v = + g_{m_N} (r_{o_N} \parallel r_{o_P}) = 20$$

$$r_{o_N} = \frac{1}{\lambda_n \frac{I_{SS}}{2}} = \frac{1}{0.1 \frac{0.556 \times 10^{-3}}{2}} = 36 \text{ k}\Omega$$

$$r_{o_P} = \frac{1}{\lambda_p \frac{I_{SS}}{2}} = 18 \text{ k}\Omega$$

$$g_{m_N} (36 \text{ k} \parallel 18 \text{ k}) = 20 \Rightarrow g_{m_N} = 1.667 \text{ mS}$$

$$\Rightarrow g_{m_N} = \frac{2 I_{D_N}}{(V_{GS} - V_{TH})_{NMOS}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{NMOS}} \Rightarrow$$

$$(V_{GS} - V_{TH})_{NMOS} = 0.333 \text{ V}$$

$$(V_{GS} - V_{TH})_{NMOS} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_{NMOS}}} \Rightarrow \left(\frac{W}{L}\right)_{1/2} = 50$$

$$V_N = V_{in,CM} - V_{TH,n} = 1 - 0.5 = 0.5 \text{ V}$$

$$\rightarrow |V_{G_3}| - |V_{TH,p}| = 1.3 - 0.4 = 0.9 \text{ V}$$

$$\rightarrow 0.9 = \sqrt{\frac{I_{SS}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_{PMOS}}} \Rightarrow$$

$$\left(\frac{W}{L}\right)_{3,4} = 13.717$$