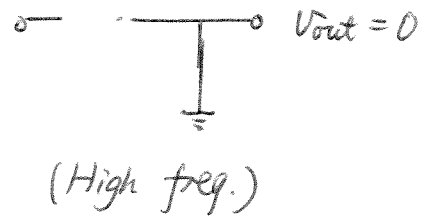
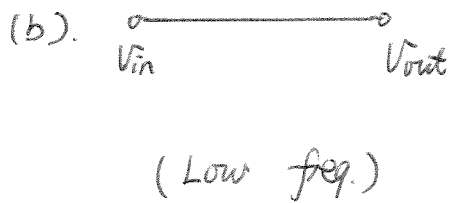
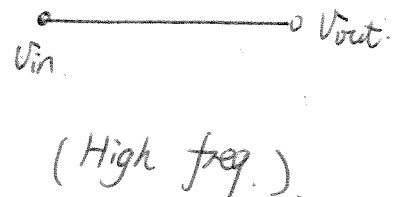
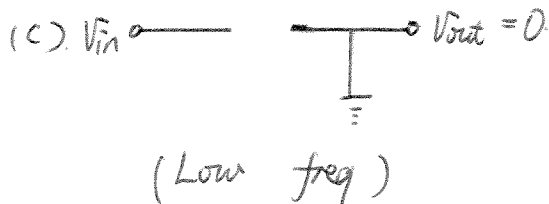


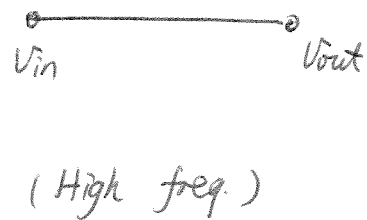
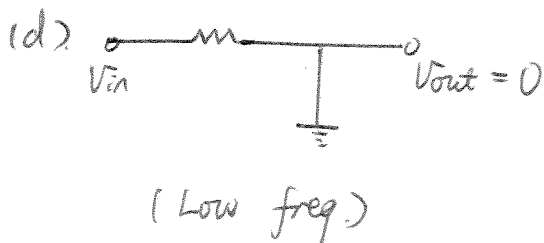
This is a high pass filter.



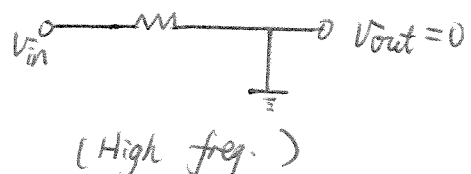
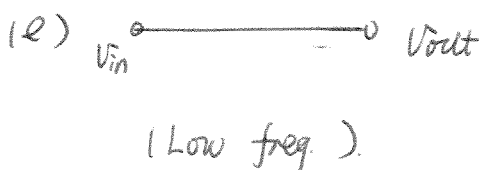
This is a low pass filter.



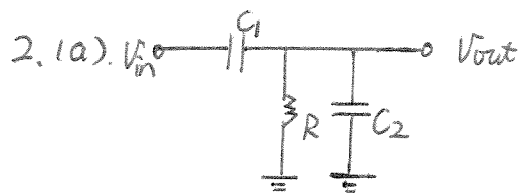
This is a high pass filter.



This is a high pass filter.



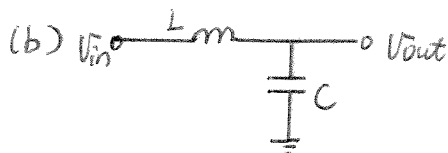
This is a low pass filter.



$$V_{out} = \frac{R // \frac{1}{sC_2}}{\frac{1}{sC_1} + R // \frac{1}{sC_2}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{C_1}{C_1 + C_2} s}{s + \frac{1}{R(C_1 + C_2)}}$$

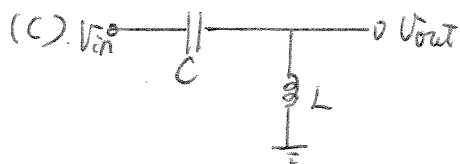
$$\text{zero} = 0; \text{ pole} = -\frac{1}{R(C_1 + C_2)}$$



$$V_{out} = \frac{\frac{1}{sC}}{sL + \frac{1}{sC}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{1/(LC)}{s^2 + \frac{1}{LC}}$$

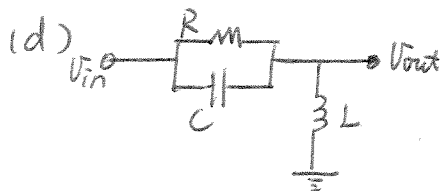
$$\text{zero: No finite zero; poles} = \pm i\sqrt{\frac{1}{LC}}$$



$$V_{out} = \frac{sL}{sL + \frac{1}{sC}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{s^2 LC}{s^2 + \frac{1}{LC}}$$

$$\text{zeros: Two zeros at } 0; \text{ poles} = \pm i\sqrt{\frac{1}{LC}}$$

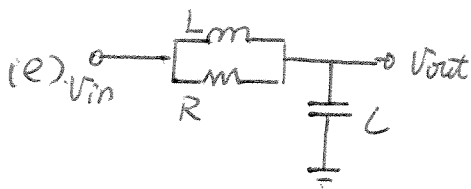


$$V_{out} = \frac{sL}{sL + R // \frac{1}{sC}} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{s(1 + \frac{1}{RC}) (RC)^2}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$\text{zeros: } 0, -\frac{1}{RC}$$

$$\text{poles} = \frac{-\frac{1}{RC} \pm \sqrt{(\frac{1}{RC})^2 - \frac{4}{LC}}}{2}$$



$$V_{out} = \frac{\frac{1}{sC}}{\frac{1}{sC} + sL // R} V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{RC} (s + \frac{R}{L})}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$\text{zero} = -\frac{R}{L}; \text{ poles} = \frac{-\frac{1}{RC} \pm \sqrt{(\frac{1}{RC})^2 - \frac{4}{LC}}}{2}$$

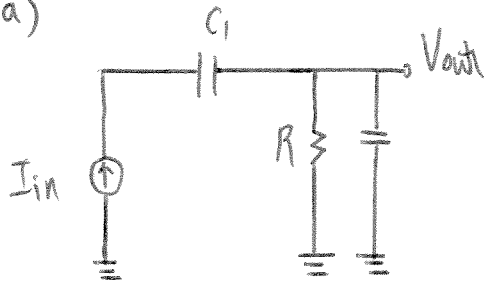
B.

Since $\frac{V_{out}}{V_{in}} = \frac{1}{(s+a)(s+b)}$, where a and b

are real and positive, the transfer function contains no finite zero and two real poles on the left hand plane. But, after reviewing Problem #2 we discover that NONE of the networks yield this case.

4.

a)

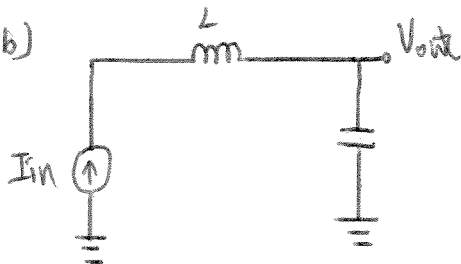


$$\frac{V_{out}}{I_{in}} = \frac{1/C_2}{s + 1/(RC_2)}$$

Zero: No finite zero

Pole: $-1/(RC_2)$

b)

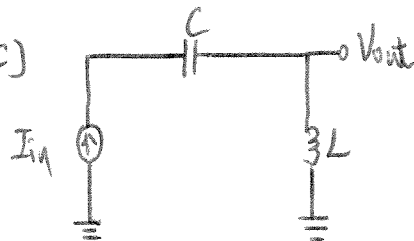


$$\frac{V_{out}}{I_{in}} = \frac{1}{sC}$$

Zero: No finite zero

Pole: 0

c)

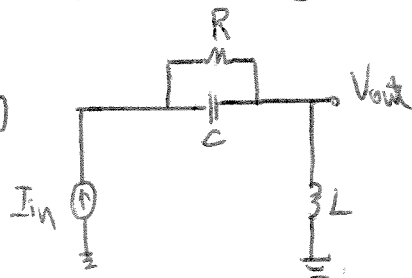


$$\frac{V_{out}}{I_{in}} = sL$$

Zero: 0

Pole: No finite pole

d)

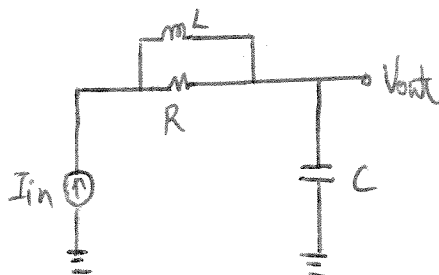


$$\frac{V_{out}}{I_{in}} = sL$$

Zero: 0

Pole: No finite pole

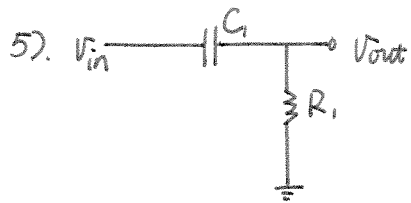
e)



$$\frac{V_{out}}{I_{in}} = \frac{1}{sC}$$

Zero: No finite zero

Pole: 0



$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 + \frac{1}{sC_1}} = \frac{s}{s + \frac{1}{R_1 C_1}}$$

zero = 0; pole = $-\frac{1}{R_1 C_1}$.

$$\frac{dP}{dC_1} = \frac{1}{(R_1 C_1)^2} \cdot \frac{1}{R_1} = \frac{1}{R_1 C_1^2}.$$

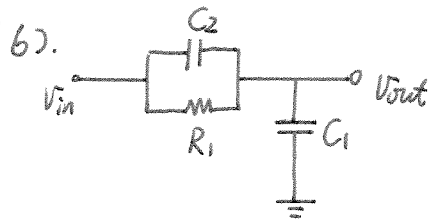
$$S_{C_1}^P = \frac{\frac{dP}{P}}{\frac{dC_1}{C_1}} = \frac{dP}{dC_1} \cdot \frac{C_1}{P} = -\frac{1}{R_1 C_1^2} \cdot C_1 \cdot (R_1 C_1) = -1.$$

Similarly

$$S_{R_1}^P = -1.$$

As for the sensitivity of zero, since the zero is at 0, which

is independent of R_1 and C_1 , $S_{R_1}^Z = S_{C_1}^Z = 0$.



$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + R_1 // \frac{1}{sC_2}}$$

$$= \frac{1}{R_1 C_2} \cdot \frac{s + \frac{1}{R_1 C_2}}{R_1 (C_1 + C_2) \left[s + \frac{1}{R_1 (C_1 + C_2)} \right]}$$

$$\text{zero} = -\frac{1}{R_1 C_2}, \quad \text{pole} = -\frac{1}{R_1 (C_1 + C_2)}$$

$$\frac{dP}{dR_1} = [R_1 (C_1 + C_2)]^{-2} \cdot (C_1 + C_2) = -\frac{C_1 + C_2}{R_1 (C_1 + C_2)} \cdot P = -\frac{P}{R_1}$$

$$S_{R_1}^P = \frac{\frac{dP}{P}}{\frac{dR_1}{R_1}} = \frac{dP}{dR_1} \cdot \frac{R_1}{P} = -1.$$

$$\frac{dP}{dC_1} = [R_1 (C_1 + C_2)]^{-2} \cdot R_1 = -\frac{P}{C_1 + C_2}$$

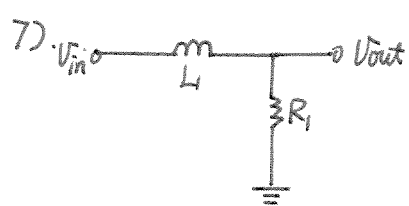
$$S_{C_1}^P = \frac{\frac{dP}{P}}{\frac{dC_1}{C_1}} = \frac{dP}{dC_1} \cdot \frac{C_1}{P} = -\frac{C_1}{C_1 + C_2}$$

Conversely,

$$S_{C_2}^P = -\frac{C_2}{C_1 + C_2}.$$

From Problem 5).

$$S_{R_1}^Z = S_{C_2}^Z = -1.$$



$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 + Ls} = \frac{R_1/L}{s + R_1/L}$$

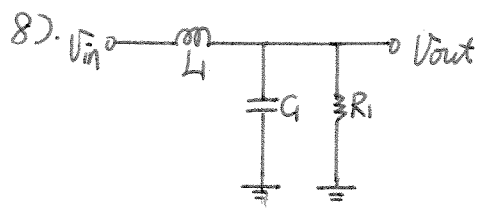
$$\text{pole} = -\frac{R_1}{L}$$

$$dp = \frac{\partial p}{\partial R_1} \cdot dR_1 + \frac{\partial p}{\partial L} dL = -\frac{1}{L} dR_1 + \frac{R}{L^2} dL$$

$$\Rightarrow \frac{dp}{p} = \frac{dR_1}{R_1} - \frac{dL}{L}$$

$$\left| \frac{dp}{p} \right| \leq 5\%, \text{ and } \left| \frac{dR_1}{R_1} \right| \leq 3\%$$

$$\Rightarrow \left| \frac{dL}{L} \right| \leq 2\%$$



$$\begin{aligned} a). V_{out} &= V_{in} \cdot \frac{R_1 // \frac{1}{C_1 s}}{R_1 // \frac{1}{C_1 s} + sL_1} \\ &= V_{in} \cdot \frac{\frac{R_1}{R_1 C_1 s + 1}}{\frac{R_1}{R_1 C_1 s + 1} + sL_1} \end{aligned}$$

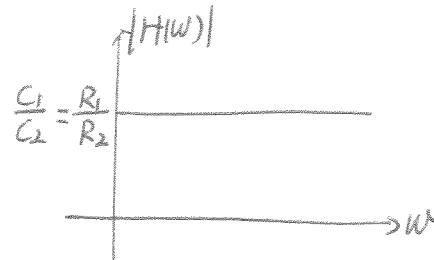
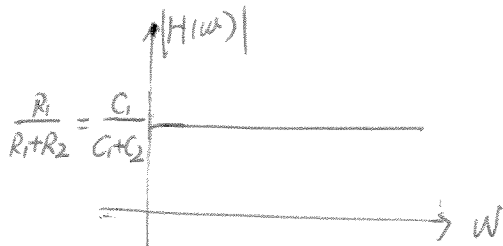
$$\frac{V_{out}}{V_{in}} = \frac{R_1}{R_1 C_1 L_1 s^2 + L_1 s + R_1} = \frac{1}{L_1 C_1} \cdot \frac{1}{s^2 + \frac{1}{R_1 C_1} s + \frac{1}{L_1 C_1}}$$

$$b). \text{poles} = \frac{-\frac{1}{R_1 C_1} \pm \sqrt{\left(\frac{1}{R_1 C_1}\right)^2 - \frac{4}{L_1 C_1}}}{2}$$

$$\text{For them to be real} \Rightarrow \left(\frac{1}{R_1 C_1}\right)^2 - \frac{4}{L_1 C_1} \geq 0$$

$$\Rightarrow \frac{1}{R_1 C_1} \geq \frac{2}{\sqrt{L_1 C_1}}$$

9). If the zero and pole coincide, they will neutralize each other, and also render the transfer function flat.



$$10). \quad H(s) = \frac{\alpha s^2 + \beta s + \gamma}{s^2 + \frac{w_n}{Q}s + w_n^2}$$

$$p_{1,2} = -\frac{w_n}{2Q} \pm jw_n \sqrt{1 - \frac{1}{4Q^2}}$$

$$\text{If } Q = \frac{1}{2}, \text{ then } p_{1,2} = -\frac{w_n}{2Q}.$$

11.

$$|H(j\omega)|^2 = \frac{Y^2}{(\omega_n^2 - \omega^2)^2 + \left(\frac{\omega_n}{Q}\omega\right)^2}$$

No peaking means no local minimum for $(\omega_n^2 - \omega^2)^2 + \left(\frac{\omega_n}{Q}\omega\right)^2$, which is also known as $D(\omega)$.

A local min exists if $\frac{\partial D(\omega)}{\partial \omega} = 0$.

$$\frac{\partial D(\omega)}{\partial \omega} = \left(\frac{\partial D(\omega)}{\partial \omega^2} \right) \left(\frac{\partial \omega^2}{\partial \omega} \right), \quad \frac{\partial D(\omega)}{\partial \omega^2} = -2(\omega_n^2 - \omega^2) + \left(\frac{\omega_n}{Q}\right)^2$$

$$\frac{\partial \omega^2}{\partial \omega} = 2\omega, \text{ so } \frac{\partial D(\omega)}{\partial \omega} = 2\omega \left[-2(\omega_n^2 - \omega^2) + \left(\frac{\omega_n}{Q}\right)^2 \right] = 0$$

$$\text{Solving for } \omega, \text{ we have } \omega = 0, \pm \sqrt{\omega_n^2 - \frac{1}{2} \left(\frac{\omega_n}{Q}\right)^2}$$

Will bring $D(\omega)$ to its min value.

At $\omega=0$, we have the DC value of the transfer function.

However if $Q^2 < \frac{1}{2}$ or $Q < \frac{1}{\sqrt{2}}$, $\omega_n^2 - \frac{1}{2} \left(\frac{\omega_n}{Q}\right)^2$ becomes negative, which is not physical. Therefore, there is no peaking for $Q < \frac{1}{\sqrt{2}}$. And at $Q = \frac{1}{\sqrt{2}}$, we have $\omega = \pm 0$, which corresponding to the DC value of the transfer function, not peaking. Therefore, the only option left is for $Q > \frac{1}{\sqrt{2}}$, and that is the condition for peaking.

$$12). |H(j\omega)|^2 = \frac{\gamma^2}{(\omega_n^2 - \omega^2)^2 + \left(\frac{\omega_n}{Q}\omega\right)^2}$$

If $Q > \sqrt{2}/2$, it will peak at $\omega_0 = \omega_n \sqrt{1 - 1/(2Q)^2}$

$$H(j\omega) = \frac{\gamma}{\sqrt{(\omega_n^2 - \omega^2)^2 + \left(\frac{\omega_n}{Q}\omega\right)^2}}$$

$$\Rightarrow H(j\omega_0) = \frac{\gamma}{\sqrt{[\omega_n^2 - (\omega_n \sqrt{1 - \frac{1}{2Q^2}})^2]^2 + \left(\frac{\omega_n}{Q} \omega_n \sqrt{1 - \frac{1}{2Q^2}}\right)^2}}$$

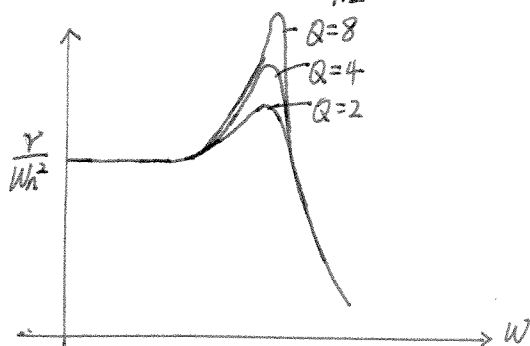
$$= \frac{\gamma}{\sqrt{\left(\omega_n^2 \cdot \frac{1}{2Q^2}\right)^2 + \frac{\omega_n^4}{Q^2} \left(1 - \frac{1}{2Q^2}\right)}}$$

$$= \frac{\gamma}{\sqrt{\frac{\omega_n^4}{4Q^4} + \frac{\omega_n^4}{Q^2} - \frac{\omega_n^4}{2Q^4}}}$$

$$= \frac{Q\gamma}{\omega^2 \sqrt{1 - \frac{1}{4Q^2}}}$$

Normalize to passband $\Rightarrow \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}}$

$Q=2$, peak = $\frac{2}{\sqrt{1 - \frac{1}{4 \times 2^2}}} = 2.07$; $Q=4$, peak = 4.03; $Q=8$, peak = 8.02



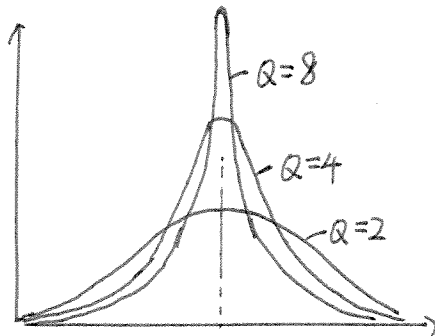
$$13). \quad H(s) = \frac{\beta s}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}, \quad H(j\omega) = \frac{j\beta\omega}{\omega_n^2 + j\frac{\omega_n}{Q}\omega - \omega^2}$$

$$|H(j\omega)| = \frac{\beta\omega}{\sqrt{(\omega_n^2 - \omega^2)^2 + \left(\frac{\omega_n}{Q}\omega\right)^2}}$$

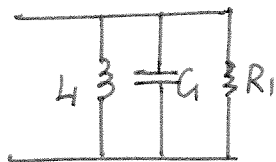
At $\omega = \omega_n$,

$$|H(j\omega_n)| = \frac{\beta\omega_n}{\frac{\omega_n}{Q}\omega} = \frac{Q}{\omega_n} \beta.$$

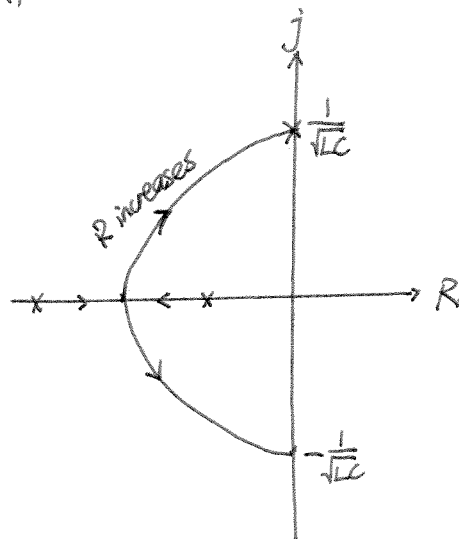
So if we normalize to β , we get $\frac{Q}{\omega_n}$.



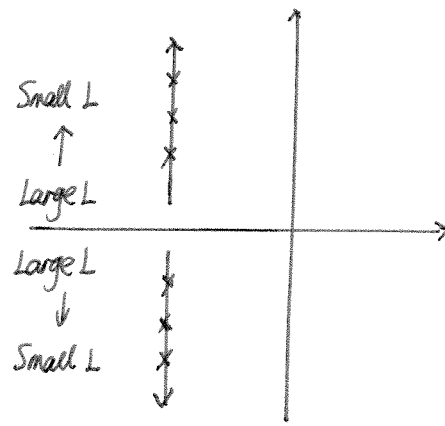
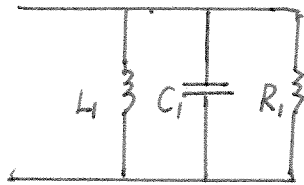
14).



Assume R is
never negative



15).



16.

$$1 \text{ dB peaking} \Rightarrow \frac{Q^2}{(1 - \frac{1}{4Q^2})} = (1.1)^2 = 1.21$$

$$Q^2 = 1.21(1 - \frac{1}{4Q^2}) \Rightarrow 4Q^4 - 4(1.1)^2 Q^2 + (1.1)^2 = 0$$

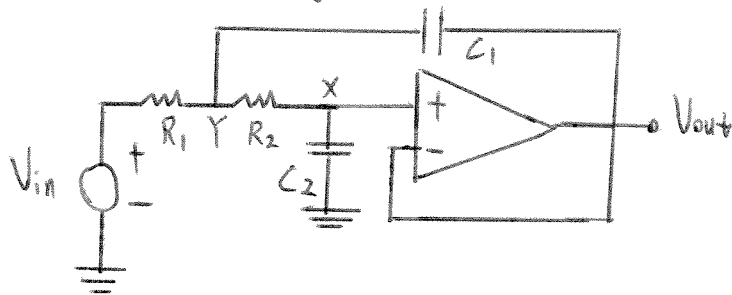
$$Q^2 = 0.85704, 0.35296, Q = 0.925765, 0.59410$$

$$Q = 0.925765, \text{ since } Q > \frac{1}{\sqrt{2}} \text{ for peaking}$$

$$Q = \frac{\omega_n}{\beta} = \frac{RC}{\sqrt{LC}} = R\sqrt{\frac{C}{L}} = 0.925765$$

17.

Sallen and Key filter



$$H(s) = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1 + R_2) C_2 s + 1}, \quad Q = \frac{1}{R_1 + R_2} \sqrt{\frac{R_1 R_2 C_1}{C_2}}$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$H(s) = \left(s^2 + \frac{(R_1 + R_2) C_2}{R_1 R_2 C_1 C_2} s + \frac{1}{R_1 R_2 C_1 C_2} \right)^{-1}$$

$$P_{1,2} = -\frac{(R_1 + R_2)}{R_1 R_2 C_1} \pm \sqrt{\left(\frac{R_1 + R_2}{R_1 R_2 C_1} \right)^2 - \frac{4}{R_1 R_2 C_1 C_2}}$$

$$P_{1,2} = -\frac{1}{2(R_1 // R_2) C_1} \pm \sqrt{\left(\frac{1}{(R_1 // R_2) C_1} \right)^2 - \frac{4}{R_1 C_1 R_2 C_2}}$$

Assuming $\frac{4}{R_1 C_1 R_2 C_2} > \frac{1}{[(R_1 // R_2) C_1]^2}$

$$P_{1,2} = -\frac{1}{2(R_1 // R_2) C_1} \pm j 2 \sqrt{\frac{1}{R_1 C_1 R_2 C_2} - \frac{1}{4[(R_1 // R_2) C_1]^2}}$$

17.

a) $R_1: 0 \rightarrow \infty$

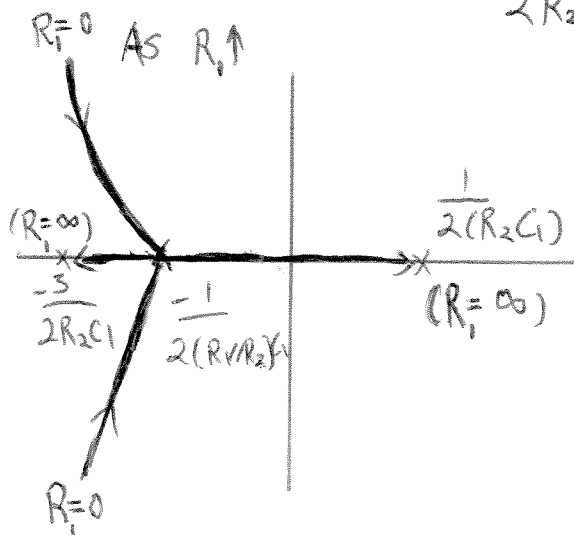
When $R_1 = 0$, Poles are at $\pm\infty$, so no finite poles. As $R_1 \uparrow$, $\frac{1}{R_1 C_1 R_2 C_2}$ approaches 0, and

$\frac{1}{4[(R_1 // R_2) C_1]^2}$ approaches $\frac{1}{4[R_2 C_1]^2}$. There exists

a R_1 such that $\frac{1}{R_1 C_1 R_2 C_2} = \frac{1}{4[(R_1 // R_2) C_1]^2} \Rightarrow$

$$P_{1,2} = -\frac{1}{2(R_1 // R_2) C_1}$$

As $R_1 \rightarrow \infty$, $P_{1,2} = -\frac{1}{2R_2 C_1} \pm \frac{1}{R_2 C_1} = -\frac{3}{2R_2 C_1}, \frac{1}{2R_2 C_1}$



17. b)

R_2 from $0 \rightarrow \infty$

When $R_2 = 0$, $P_{1,2}$ are at $\pm \infty$

$$\text{As } R_2 \uparrow, \frac{-1}{2(R_1 \parallel R_2)C_1} \rightarrow -\frac{1}{2R_1C_1}$$

$$\frac{1}{R_1C_1R_2C_2} \rightarrow 0, \text{ and } \frac{1}{4[(R_1 \parallel R_2)C_1]^2} \rightarrow \frac{1}{4[R_1C_1]^2}$$

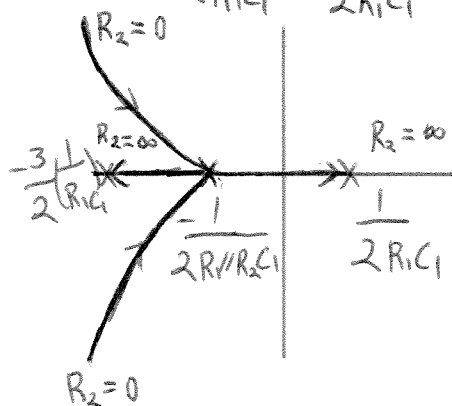
$$\text{For a certain } R_2, \frac{1}{R_1C_1R_2C_2} = \frac{1}{4[(R_1 \parallel R_2)C_1]^2}$$

$$\text{and } P_{1,2} = \frac{-1}{2(R_1 \parallel R_2)C_1}$$

Finally, when $R_2 = \infty$,

$$P_{1,2} = -\frac{1}{2R_1C_1} \pm 2\sqrt{\frac{1}{4[R_1C_1]^2}} = -\frac{1}{2R_1C_1} \pm \frac{1}{R_1C_1}$$

$$P_{1,2} = -\frac{3}{2} \frac{1}{R_1C_1}, \frac{1}{2R_1C_1}$$

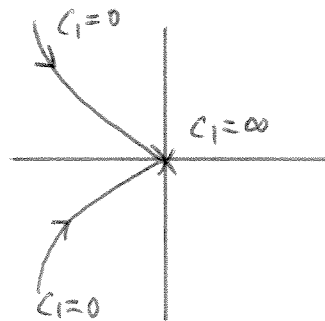


17. c)

As $C_1: 0 \rightarrow \infty$

When $C_1 = 0$, poles are at $\pm \infty$

As $C_1 \uparrow$, poles approach 0.

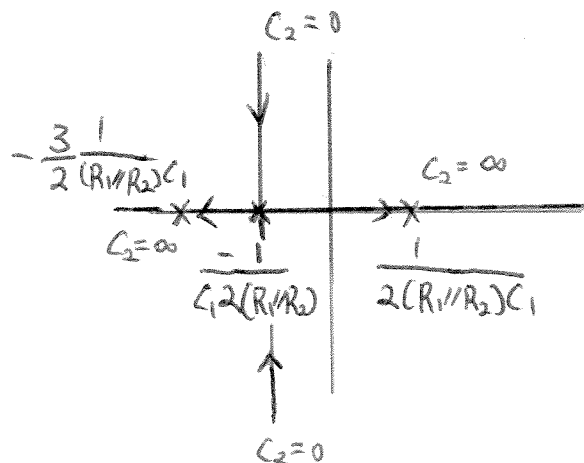


d) As $C_2: 0 \rightarrow \infty$

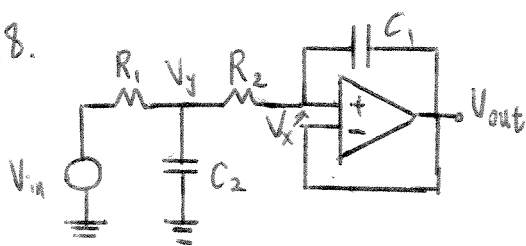
When $C_2 = 0$, poles are at $\pm \infty$

When $C_2 = \infty$, poles: $-\frac{1}{2(R_1//R_2)C_1} \pm \frac{1}{R_1//R_2 C_1} = -\frac{3}{2} \left(\frac{1}{R_1//R_2 C_1} \right)$

$+\frac{1}{2 R_1//R_2 C_1}$ (note, real part doesn't depend on C_2).



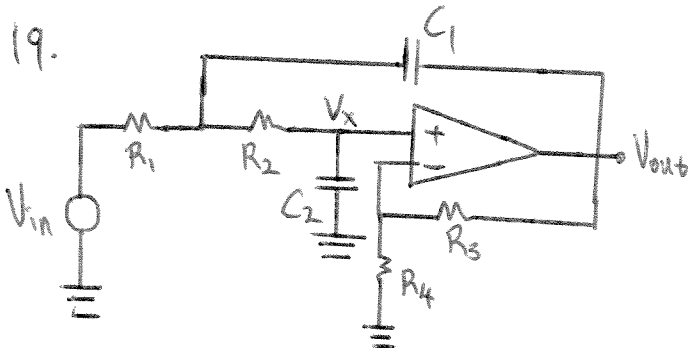
18.



Assuming an ideal Op-amp, $V_x = V_{out}$. Therefore, no current will flow thru C_1 . Moreover, since the input impedance of an Op-amp (ideal) is infinite, no current will flow thru R_2 as well, which means $V_y = V_x = V_{out}$.

$$\Rightarrow V_y = V_{out} = \frac{1/C_2 s}{R_1 + 1/C_2 s}$$

Not very useful since it's only a simple single pole lowpass filter. We can implement it with passive components, instead of op-amp.



$$K=4, \quad C_1=C_2$$

$$Q=4$$

$$K = 1 + \frac{R_3}{R_4} = 4 \Rightarrow \frac{R_3}{R_4} = 3, \quad \frac{C_1}{C_2} = 1$$

$$\frac{1}{Q} = \sqrt{\frac{R_1 C_2}{R_2 C_1}} + \sqrt{\frac{R_2 C_2}{R_1 C_1}} - \sqrt{\frac{R_1 C_1}{R_2 C_2}} \frac{R_3}{R_4}$$

$$\frac{1}{Q} = \sqrt{\frac{R_1}{R_2}} + \sqrt{\frac{R_2}{R_1}} - 3\sqrt{\frac{R_1}{R_2}} \Rightarrow \sqrt{\frac{R_2}{R_1}} - 2\sqrt{\frac{R_1}{R_2}} = \frac{1}{Q}$$

$$\frac{1}{Q} = \left(\frac{R_1}{R_2}\right)^{\frac{1}{2}} - 2\left(\frac{R_1}{R_2}\right)^{\frac{1}{2}} \Rightarrow \text{squaring both sides} \Rightarrow$$

$$\frac{1}{Q^2} = 4\left(\frac{R_1}{R_2}\right) - 4 + \left(\frac{R_1}{R_2}\right)^{-1} \Rightarrow \frac{1}{16} = 4\left(\frac{R_1}{R_2}\right) - 4 + \left(\frac{R_2}{R_1}\right)$$

$$\left(\frac{1}{16} + 4\right) \frac{R_1}{R_2} = \frac{R_1}{R_2} \left(4\frac{R_1}{R_2} + \frac{R_2}{R_1}\right) \Rightarrow 4.0625 \frac{R_1}{R_2} = 4\left(\frac{R_1}{R_2}\right)^2 + 1$$

$$4\left(\frac{R_1}{R_2}\right)^2 - 4.0625\left(\frac{R_1}{R_2}\right) + 1 = 0, \quad \frac{R_1}{R_2} = 0.41908, 0.59055$$

This leads to a negative Q.

19.

$$S_{R1}^{\theta} = -\frac{1}{2} \left[\sqrt{\frac{R_1 C_2}{R_2 C_1}} - \sqrt{\frac{R_2 C_2}{R_1 C_1}} - (K-1) \sqrt{\frac{R_1 C_1}{R_2 C_2}} \right] \theta$$

$$S_{R1}^{\theta} = -\frac{1}{2} \left[\sqrt{0.41908} - \sqrt{1/0.41908} - 3\sqrt{0.41908} \right] 4$$

$$S_{R1}^{\theta} = 5.68$$

20.

$$H(s) = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1 + R_2) C_2 s + 1}$$

$$Q = \frac{1}{R_1 + R_2} \sqrt{R_1 R_2 \frac{C_1}{C_2}}, \quad \omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}} = 1.1 \Rightarrow Q^2 = (1.1)^2 \left(1 - \frac{1}{4Q^2}\right)$$

$$\Rightarrow 3.3058 Q^4 - 4Q^2 + 1 = 0$$

$$Q^2 = 0.85704, 0.35296,$$

$$Q = \pm 0.925765, \pm 0.5941$$

$$\text{In order to peak, } Q > \frac{1}{\sqrt{2}} \Rightarrow Q = 0.925765$$

$$\Rightarrow \frac{1}{R_1 + R_2} \sqrt{R_1 R_2 \frac{C_1}{C_2}} = 0.925765$$

$$\text{let } \frac{C_1}{C_2} = 1 \Rightarrow \frac{1}{R_1 + R_2} \sqrt{R_1 R_2} = 0.925765$$

$$\Rightarrow R_1 R_2 = (0.925765)^2 (R_1 + R_2)^2$$

$$\frac{R_1 R_2}{R_1 + R_2} = R_1 // R_2 = 0.85704 (R_1 + R_2)$$

$$R_1 // R_2 = 0.85704 (R_1 + R_2)$$

$$\text{only if } \frac{C_1}{C_2} = 1$$

21.

$$S_{R_1}^Q = 2, \quad C_2 = C_1, \quad Q = f\left(\sqrt{\frac{R_2}{R_1}}\right)$$

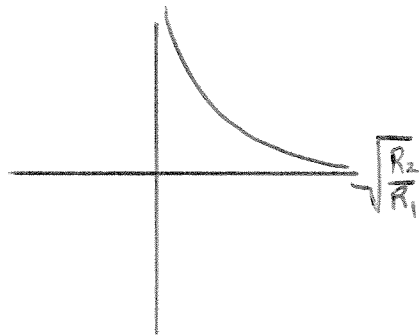
Range of Q and $\sqrt{R_2/R_1}$

$$S_{R_1}^Q = -\frac{1}{2} \left[\sqrt{\frac{R_1 C_2}{R_2 C_1}} - \sqrt{\frac{R_2 C_2}{R_1 C_1}} - (K-1) \sqrt{\frac{R_1 C_1}{R_2 C_2}} \right] Q$$

$$S_{R_1}^Q = -\frac{1}{2} + Q \sqrt{\frac{R_2 C_2}{R_1 C_1}} \Rightarrow 2 = -\frac{1}{2} + Q \sqrt{\frac{R_2}{R_1}}$$

$$\Rightarrow 2.5 = Q \sqrt{\frac{R_2}{R_1}}, \quad Q = \frac{2.5}{\sqrt{R_2/R_1}}$$

Q



($\frac{R_2}{R_1}$ can't be negative)

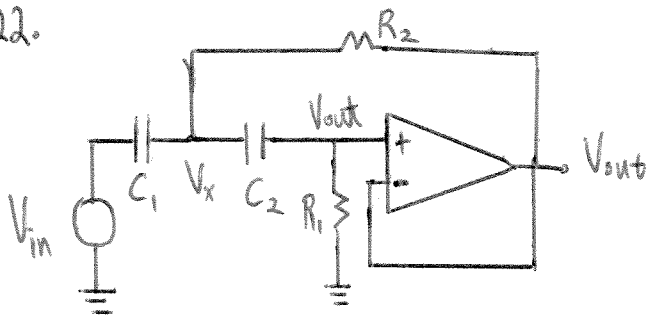
Range:

$$0 < Q < \infty$$

$$0 < \sqrt{\frac{R_2}{R_1}} < \infty$$

If the transfer function does not want to experience peaking, then $0 < Q \leq \frac{1}{\sqrt{2}} \Rightarrow 3.5356 \leq \sqrt{\frac{R_2}{R_1}} < \infty$

22.



Assuming an ideal
Op amp.

$$1) (V_x - V_{in})C_1 s + (V_x - V_{out})\left(C_2 s + \frac{1}{R_2}\right) = 0, \text{ nodal equation at } V_x.$$

$$2) (V_x - V_{out})C_2 s - \frac{V_{out}}{R_1} = 0, \text{ nodal equation at } V_{out}.$$

$$2) \Rightarrow V_x = V_{out} \left[\frac{C_2 s + 1/R_1}{C_2 s} \right] \quad (A) \quad \text{The stuff in the bracket becomes "A"}$$

$$1) \Rightarrow (AV_{out} - V_{in})C_1 s + (AV_{out} - V_{out})\left[C_2 s + \frac{1}{R_2}\right] = 0$$

$$\Rightarrow AV_{out}C_1 s + V_{out}(A-1)(C_2 s + 1/R_2) = V_{in}C_1 s$$

$$A-1 = \frac{1}{R_1 C_2 s}, \quad A = \frac{C_2 s + 1/R_1}{C_2 s}$$

Substitute (A-1) and A into 1) $\Rightarrow$

$$\left(\frac{C_2 s + 1/R_1}{C_2 s}\right)C_1 s V_{out} + \left(C_2 s + \frac{1}{R_2}\right)\frac{1}{R_1 C_2 s} V_{out} = V_{in}C_1 s$$

22.

$$\frac{V_{out}}{V_{in}} = \frac{C_1 S}{\frac{C_1}{C_2} \left(C_2 S + \frac{1}{R_1} \right) + \frac{1}{R_1 C_2 S} \left(C_2 S + \frac{1}{R_2} \right)}$$

Rearranging

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + \left(\frac{C_1 + C_2}{C_2 R_1 C_1} \right) s + \frac{1}{R_2 C_2 R_1 C_1}}$$

$$\omega_n^2 = \frac{1}{R_2 C_2 R_1 C_1}, \quad \frac{\omega_n}{Q} = \frac{C_1 + C_2}{C_2 R_1 C_1}$$

$$\omega_n = \frac{1}{\sqrt{R_2 C_2 R_1 C_1}}, \quad Q = \sqrt{\frac{C_2 C_1 R_1}{R_2}} \left(\frac{1}{C_1 + C_2} \right)$$

23.

$$Q = \frac{1}{C_1 + C_2} \sqrt{\frac{C_2 C_1 R_1}{R_2}} \Rightarrow \frac{1}{Q} = (C_1 + C_2) \sqrt{\frac{R_2}{C_2 C_1 R_1}}$$

$$1) \frac{d}{dQ} \left[\frac{1}{Q} \right] = -\frac{1}{Q^2} \Rightarrow d \left[\frac{1}{Q} \right] = -\frac{1}{Q^2} dQ$$

$$2) \frac{d \left[\frac{1}{Q} \right]}{dR_2} = \frac{1}{2} \frac{C_1 + C_2}{\sqrt{C_2 C_1 R_1 R_2}} \Rightarrow d \left[\frac{1}{Q} \right] = \frac{1}{2} \frac{C_1 + C_2}{\sqrt{C_2 C_1 R_1 R_2}} dR_2$$

Equating 1) and 2) and multiple 2) by $\frac{R_2}{R_2}$

$$-\frac{dQ}{Q^2} = \frac{1}{2} \frac{(C_1 + C_2) R_2}{\sqrt{C_2 C_1 R_1 R_2}} \frac{dR_2}{R_2}$$

$$\frac{dQ}{Q} / \frac{dR_2}{R_2} = -\frac{Q}{2} (C_1 + C_2) \sqrt{\frac{R_2}{C_1 C_2 R_1}}$$

$$S_{R_2}^Q = -\frac{Q}{2} (C_1 + C_2) \sqrt{\frac{R_2}{C_1 C_2 R_1}} = -\frac{1}{2}$$

$$\frac{1}{Q} = (C_1 + C_2) \sqrt{\frac{R_2}{C_2 C_1 R_1}} = C_1 \sqrt{\frac{R_2}{C_2 C_1 R_1}} + C_2 \sqrt{\frac{R_2}{C_2 C_1 R_1}}$$

$$\frac{\partial \left(\frac{1}{Q} \right)}{\partial C_1} = \frac{1}{2} \sqrt{\frac{R_2}{C_2 R_1 C_1}} - \frac{C_2}{2 C_1} \sqrt{\frac{R_2}{C_2 R_1 C_1}}, \quad \frac{d \left(\frac{1}{Q} \right)}{dQ} = -\frac{1}{Q^2}$$

$$\text{Rearranging} \Rightarrow -\frac{\partial Q}{Q^2} = \frac{\partial C_1}{C_1} \left(\frac{C_1 - C_2}{2} \sqrt{\frac{R_2}{C_2 R_1 C_1}} \right)$$

$$\frac{\partial Q}{Q} / \frac{\partial C_1}{C_1} = S_{C_1}^Q = -Q \left(\frac{C_1 - C_2}{2} \sqrt{\frac{R_2}{C_2 R_1 C_1}} \right)$$

23.

Similarly:

$$S_{C_2}^R = -Q \left(\frac{C_2 - C_1}{2} \sqrt{\frac{R_2}{C_2 R_1 C_1}} \right)$$

$$S_{R_1}^R = Q \left(\frac{C_1 + C_2}{2} \sqrt{\frac{R_2}{C_2 C_1 R_1}} \right) = \frac{1}{2}$$

24.

$$\frac{V_{out}(s)}{V_{in}} = \frac{\alpha s^2}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2}$$

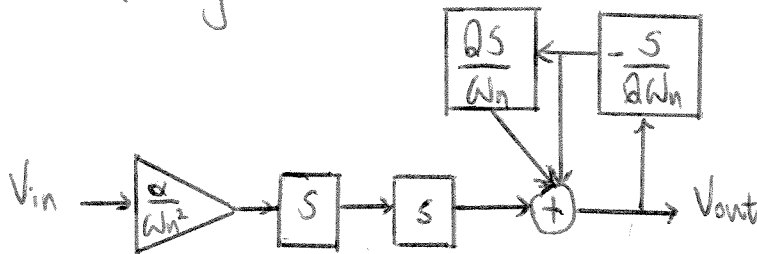
Cross-multiply.

$$V_{out} s^2 + V_{out} \frac{\omega_n s}{Q} + V_{out} \omega_n^2 = V_{in} \alpha s^2$$

Rearranging

$$V_{out} = V_{in} \frac{\alpha}{\omega_n^2} s^2 - V_{out} \frac{s^2}{\omega_n^2} - V_{out} \frac{s}{Q \omega_n}$$

Block diagram:



25.

$$Q=2, \omega_n = (2\pi)(2 \times 10^6)$$

$$R_6 = R_3, R_1 = R_2, C_1 = C_2$$

$$10\text{pF} < \text{Total } C < 1\text{nF}, \quad 1\text{K}\Omega < \text{Total } R < 50\text{K}\Omega$$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4 + R_5} \left(\frac{1}{R_1 C_1} \right), \quad \omega_n^2 = \frac{R_6}{R_3} \left(\frac{1}{R_1 R_2 C_1 C_2} \right)$$

$$\text{Since } R_6 = R_3 \Rightarrow \omega_n^2 = \left(\frac{1}{R_1 C_1} \right)^2 = (2\pi \times 2 \times 10^6)^2$$

$$\frac{1}{R_1 C_1} = 2\pi \times 2 \times 10^6 = \omega_n$$

$$Q = \frac{R_4 + R_5}{R_4} = 2 \Rightarrow R_5 = R_4$$

$$\text{Let } C_1 = C_2 = 100\text{pF}, \quad R_1 = \frac{1}{(2\pi)(2 \times 10^6)(100\text{pF})} = 795.77\Omega$$

$$\text{So } R_1 = R_2 = 795.77\Omega, \quad C_1 = C_2 = 100\text{pF}.$$

Since R_3, R_4, R_5, R_6 don't affect Q and ω_n ,
let them be 500Ω each.

$$\text{Total } R: (4)(500) + (2)(795.77) = 3.6\text{K}\Omega$$

$$\text{Total } C: 100\text{pF} + 100\text{pF} = 200\text{pF}.$$

26.

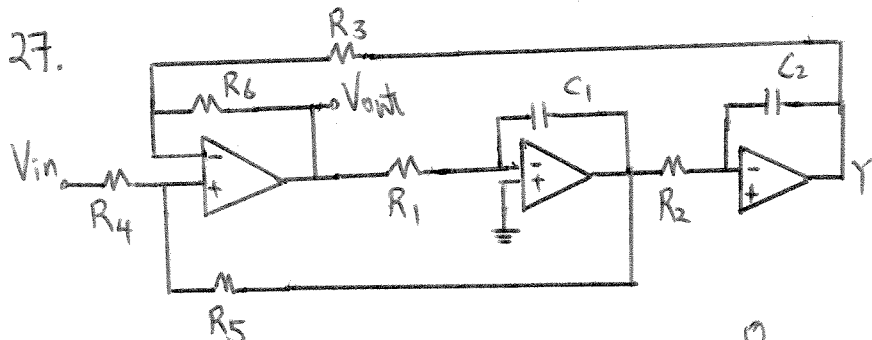
$$\frac{\omega_n}{Q} = \frac{R_4}{R_4 + R_5} \frac{1}{R_1 C_1}, \quad \omega_n^2 = \frac{R_6}{R_3} \cdot \frac{1}{R_1 R_2 C_1 C_2}$$

$$Q = \omega_n \left(\frac{R_4 + R_5}{R_4} \right) R_1 C_1, \quad \omega_n = \sqrt{\frac{R_6}{R_3} \left(\frac{1}{R_1 R_2 C_1 C_2} \right)}$$

$$Q = \sqrt{\frac{R_6}{R_3} \left(\frac{1}{R_1 R_2 C_1 C_2} \right)} \left(\frac{R_4 + R_5}{R_4} \right) R_1 C_1$$

$$Q = \sqrt{\frac{R_6}{R_3}} \sqrt{\frac{R_1 C_1}{R_2 C_2}} \left(\frac{R_4 + R_5}{R_4} \right)$$

If $R_6 = R_3$, Q doesn't depend on R_6 and R_3 ,
hence zero sensitivity.



Low pass, low freq gain of 2. $S_{R_3, R_6}^Q = 0$

$$\frac{V_Y}{V_{in}} = \left(\frac{\alpha S^2}{S^2 + \frac{\omega_n}{Q} S + \omega_n^2} \right) \left(\frac{1}{R_1 R_2 C_1 C_2 S^2} \right) \quad \text{Low pass transfer function}$$

Low freq gain: $\frac{\alpha}{\omega_n^2 R_1 R_2 C_1 C_2}$, where $\alpha = \frac{R_5}{R_4 + R_5} \left(1 + \frac{R_6}{R_3} \right)$

and $\omega_n^2 = \frac{R_6}{R_3} \left(\frac{1}{R_1 R_2 C_1 C_2} \right)$

Therefore, Low freq gain: $\frac{\alpha}{\frac{R_6}{R_3}} = \frac{R_5}{R_4 + R_5} \left(1 + \frac{R_6}{R_3} \right) \left(\frac{R_3}{R_6} \right)$

$$S_{R_3, R_6}^Q = \frac{Q}{2} \frac{|R_3 - R_6|}{1 + R_5/R_4} \sqrt{\frac{R_2 C_2}{R_3 R_6 R_1 C_1}}$$

To obtain $S_{R_3, R_6}^Q = 0$, $R_3 = R_6$, however this makes the low freq gain: $2 \left(\frac{R_5}{R_4 + R_5} \right) \neq 2$.

Therefore, it's impossible a low freq gain of 2 if $S_{R_3, R_6}^Q = 0$.

28.

Peaking: 1 dB, $R_3 = R_6$

Normalized Peak Value: $\frac{Q}{\sqrt{1 - (4Q^2)^{-1}}} = 1.01$

solving for Q^2 : $0.8570, 0.3830$ (not possible for peaking)

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4 + R_5} \left(\frac{1}{R_1 C_1} \right), \left(\frac{\omega_n}{Q} \right)^2 = \left(\frac{R_4}{R_4 + R_5} \right)^2 \left(\frac{1}{R_1 C_1} \right)^2$$

$$\omega_n^2 = \frac{R_6}{R_3} \left(\frac{1}{R_1 R_2 C_1 C_2} \right) = \frac{1}{R_1 R_2 C_1 C_2} \quad (\text{since } R_6 = R_3)$$

$$\text{Therefore, } Q^2 = \left(\frac{R_1 C_1}{R_2 C_2} \right) \left(\frac{R_4 + R_5}{R_4} \right)^2 = 0.8570$$

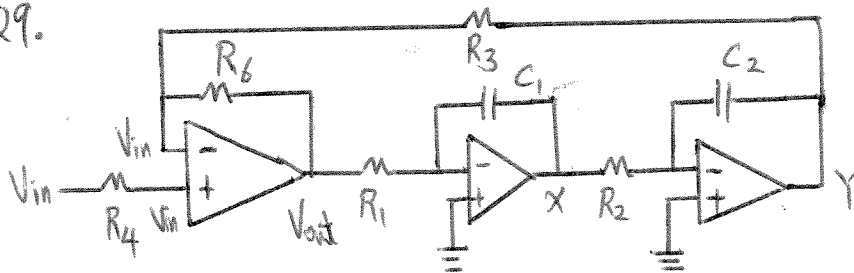
$$\text{Low pass gain: } 2 \frac{R_5}{R_4 + R_5} = \alpha \quad (\text{since } R_6 = R_3)$$

$$\text{so } \frac{\alpha}{2} = \frac{R_5}{R_4 + R_5}, \frac{R_4}{R_4 + R_5} = 1 - \frac{\alpha}{2} \Rightarrow \frac{R_4 + R_5}{R_4} = \left(1 - \frac{\alpha}{2} \right)^{-1}$$

$$\text{so } \left(\frac{R_1 C_1}{R_2 C_2} \right) \left(1 - \frac{\alpha}{2} \right)^{-2} = 0.8570, \text{ if } \alpha = 1 \Rightarrow \frac{R_1 C_1}{R_2 C_2} = 0.214.$$

However, can't go down any further without knowing more information

29.



$$V_x = -\frac{V_{out}}{R_1} \left(\frac{1}{C_1 s} \right), \quad V_Y = -\frac{V_x}{R_2} \left(\frac{1}{C_2 s} \right), \quad V_{out} = V_{in} - \frac{(V_Y - V_{in}) R_6}{R_3}$$

$$\text{Substituting } V_x \text{ into } V_Y \Rightarrow V_Y = \frac{V_{out}}{R_1} \left(\frac{1}{C_1 s} \right) \left(\frac{1}{R_2 C_2 s} \right)$$

Substituting V_Y into V_{out} and rearranging:

$$\frac{V_{out}}{V_{in}} = \frac{(R_1 C_1)(R_2 C_2) s^2 \left(1 + \frac{R_6}{R_3} \right)}{(R_1 C_1)(R_2 C_2) s^2 + \frac{R_6}{R_3}}$$

$$\frac{V_{out}}{V_{in}} = \frac{s^2 \left(1 + \frac{R_6}{R_3} \right)}{s^2 + \frac{R_6}{R_3} \left(\frac{1}{R_1 C_1 R_2 C_2} \right)}$$

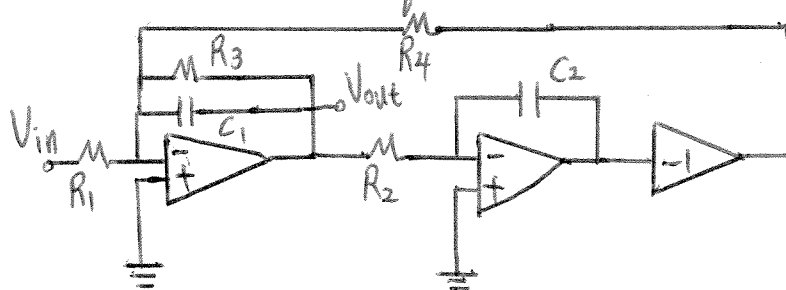
Simplifying

$$\omega_n^2 = \frac{R_6}{R_3} \left(\frac{1}{R_1 C_1 R_2 C_2} \right), \quad Q = \infty$$

$$\alpha = \left(1 + \frac{R_6}{R_3} \right)$$

30.

TOW-Thomas Biquad:



$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}, \quad Q^{-1} = \frac{1}{R_3} \sqrt{\frac{R_2 R_4 C_2}{C_1}}$$

$$\frac{\partial \omega_n}{\partial R_2} = -\frac{1}{2} \frac{1}{R_2 \sqrt{R_2 R_4 C_1 C_2}} = -\frac{1}{2} \frac{\omega_n}{R_2}$$

$$\frac{\partial \omega_n}{\omega_n} / \frac{\partial R_2}{R_2} = S_{R_2}^{\omega_n} = -\frac{1}{2}$$

Since R_2, R_4, C_1, C_2 are equivalent in ω_n 's definition, all of their sensitivities = $-\frac{1}{2}$

Sensitivities of Q :

$$\frac{\partial Q}{\partial R_3} = \sqrt{\frac{C_1}{R_2 R_4 C_2}} \left(\frac{R_3}{R_3} \right) \Rightarrow \frac{\partial Q}{Q} = \frac{\partial R_3}{R_3} \Rightarrow S_{R_3}^Q = 1$$

$$\frac{\partial Q}{\partial C_1} = \frac{1}{2} R_3 \left(\frac{C_1}{R_2 R_4 C_2} \right)^{-\frac{1}{2}} \left(\frac{1}{R_2 R_4 C_2} \right) \frac{C_1}{C_1} \Rightarrow \frac{\partial Q}{Q} = \frac{1}{2} \frac{\partial C_1}{C_1} \Rightarrow S_{C_1}^Q = \frac{1}{2}$$

$$\frac{\partial Q}{\partial R_2} = -\frac{1}{2} R_3 \left(\frac{C_1}{R_2 R_4 C_2} \right)^{-\frac{1}{2}} \frac{C_1}{R_4 C_2} \left(\frac{1}{R_2^2} \right) \Rightarrow \frac{\partial Q}{Q} = -\frac{1}{2} \frac{\partial R_2}{R_2} \Rightarrow S_{R_2}^Q = -\frac{1}{2}$$

30.

Since R_2, R_4 and C_2 are equivalent in the expression

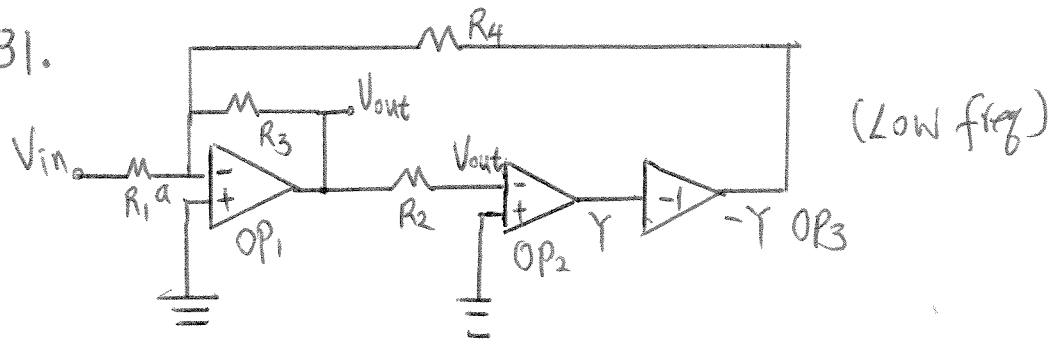
$$S_{R_2, R_4, C_2}^Q = -\frac{1}{2}$$

$$\text{So, } S_{R_2, R_4, C_1, C_2}^{W_1} = -\frac{1}{2}, \quad S_{R_1, R_3}^{W_1} = 0$$

$$S_{R_2, R_4, C_2}^Q = -\frac{1}{2}, \quad S_{C_1}^Q = \frac{1}{2}, \quad S_{R_3}^Q = 1$$

$$S_{R_1}^Q = 0$$

31.



V_{out} equals zero because of OP2's negative feedback.

Likewise, V_a equals to zero as well.

So, summing all the currents thru R_3 , we have

$$-\left(\frac{0 - V_Y}{R_4} + \frac{V_{in}}{R_1}\right) R_3 = V_{out} = 0$$

$$\Rightarrow \frac{V_{in}}{R_1} = \frac{V_Y}{R_4} \Rightarrow \frac{V_Y}{V_{in}} = \frac{R_4}{R_1}$$

32.

$$\frac{V_Y}{V_{in}} = \frac{R_3 R_4}{R_1} \left(\frac{1}{R_2 R_3 R_4 C_1 C_2 S^2 + R_2 R_4 C_2 S + R_3} \right)$$

$$\omega_n = (2\pi)(10 \text{ MHz}), \quad R_3 = 1\text{K}, \quad R_2 = R_4, \quad C_1 = C_2$$

$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}, \quad Q = \frac{1}{R_3} \sqrt{\frac{R_2 R_4 C_2}{C_1}}$$

$$\text{Peaking: } 1\text{dB}$$

$$\frac{Q}{\sqrt{1 - (4Q^2)^{-1}}} = 1.1, \quad Q^2 = 0.8570$$

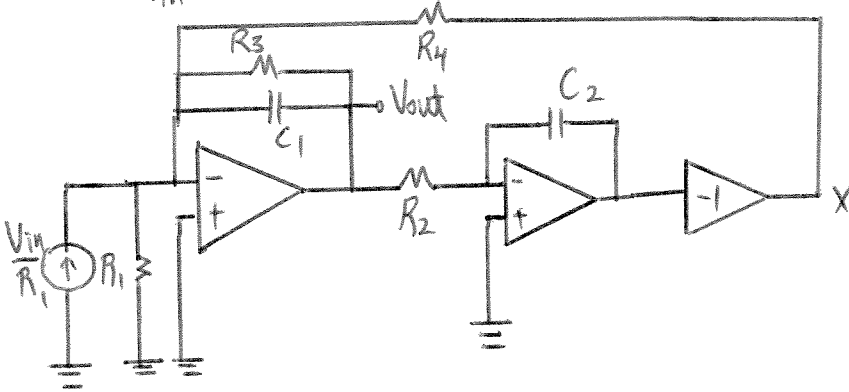
$$\omega_n = \frac{1}{\sqrt{(R_2 C_1)^2}} = (2\pi)(10 \times 10^6) \Rightarrow \frac{1}{R_2 C_1} = (2\pi)(10 \times 10^6)$$

$$\frac{1}{Q} = \frac{1}{1000} \sqrt{R_2^2} = \frac{1}{Q} = \frac{R_2}{1000} \Rightarrow \begin{matrix} R_2 = 1166.860 \text{ ohm} \\ R_2 = 1.2 \text{ K}\Omega \end{matrix}$$

$$\text{Solving for } C_1 \text{ we have: } C_1 = 13.64 \text{ pf.}$$

33.

$$\frac{V_Y}{V_{in}} = \frac{R_3 R_4}{R_1} \left(\frac{1}{R_2 R_3 R_4 C_1 C_2 S^2 + R_2 R_4 C_2 S + R_3} \right)$$



When R_1 and V_{in} are replaced with its Norton equivalent, we see that the "upper" terminal of R_1 is at virtual ground. Since R_1 's two terminals are at the same potential, no current will flow through it, therefore it can be seen as an open. So R_1 is not in the signal path, and therefore will not affect the frequency response. However, since its magnitude is embedded in the Norton current source, it will affect the DC gain.

34.

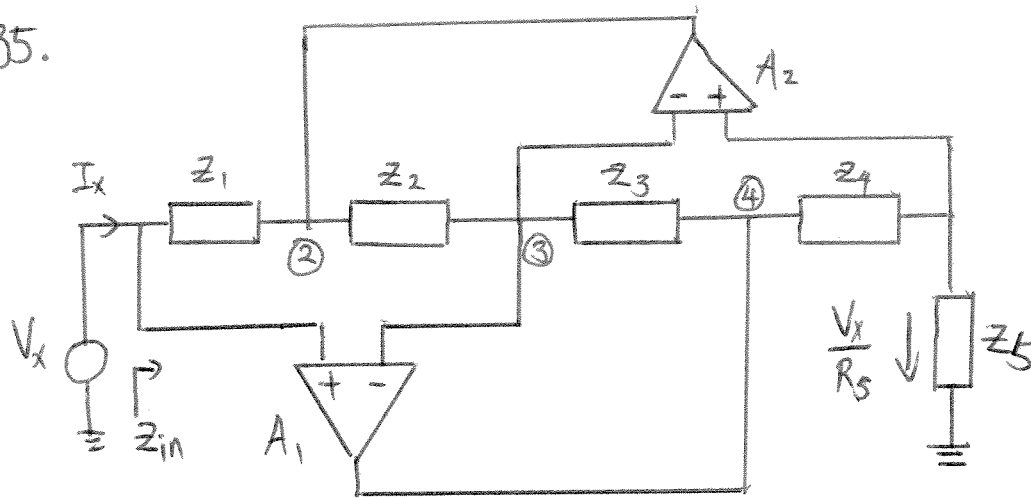
$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \quad (\text{for circuit diagram, please refer to Problem \# 35})$$

For Z_{in} to be inductive, the following combinations will work.

<u>1</u>	<u>2</u>	<u>3</u>
$Z_5 = R$	$Z_5 = R$	$Z_5 = R$
$Z_4 = R$	$Z_4 = C$	$Z_4 = C$
$Z_3 = R$	$Z_3 = R$	$Z_3 = R$
$Z_2 = C$	$Z_2 = C$	$Z_2 = R$
$Z_1 = R$	$Z_1 = C$	$Z_1 = R$

Any other combination will result in DC path blockage at a node. Moreover, in #2 it's assumed that the input can provide a DC bias.

35.



$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

For Z_{in} to be Capacitive, the following combinations can be used.

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>
$Z_5 = C$	$Z_5 = C$	$Z_5 = R$	$Z_5 = R$	$Z_5 = R$
$Z_4 = R$	$Z_4 = R$	$Z_4 = R$	$Z_4 = R$	$Z_4 = C$
$Z_3 = R$	$Z_3 = R$	$Z_3 = R$	$Z_3 = C$	$Z_3 = C$
$Z_2 = R$	$Z_2 = C$	$Z_2 = R$	$Z_2 = R$	$Z_2 = R$
$Z_1 = R$	$Z_1 = C$	$Z_1 = C$	$Z_1 = R$	$Z_1 = C$

Any other combination results in a DC path blockage at a node. Moreover, in # 2, 3, 5, it is assumed that the input node will produce a DC bias.

36.

$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

$$Z_5 = R_x + \frac{1}{Cs}, \quad Z_4 = R_x, \quad Z_3 = R_x, \quad Z_2 = R_x,$$

$$Z_1 = \frac{1}{Cs}$$

$$Z_{in} = \frac{\frac{R_x}{Cs} (R_x + \frac{1}{Cs})}{R_x^2} = \frac{1}{Cs R_x} (R_x + \frac{1}{Cs})$$

$$Z_{in} = \frac{1}{Cs} + \frac{1}{C^2 s^2 R_x}$$

$$V_{out} = \frac{V_{in} [s^2 [C^2 R_x] + Cs]}{[s^2 [C^2 R_x] + sC + R_1 [s^3 C^3 R_x]]}$$

$$\frac{V_{out}}{V_{in}} = \frac{sC R_x + 1}{s^2 R_1 R_x C^2 + sC R_x + 1}$$

37.

$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \quad (\text{for circuit diagram, please refer to problem \# 35})$$

Let Z_5 be a capacitor, Z_2 and Z_4 be large resistors and Z_1 and Z_3 be small resistors compared to Z_2 and Z_4 .

For example, let Z_1 and Z_3 equal 50Ω and Z_2 and Z_4 equal $5k\Omega$. Then there's a $(100)^2 = 10000$ multiplication factor onto C_5 .

38.

Butterworth filter: roll-off of 1dB @ $\omega = 0.9\omega_0$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (0.9)^{2n}}} = 0.9 \Rightarrow 2n = \frac{\log(0.2345679)}{\log(0.9)}$$

$$n = 6.88$$

So we need a 7th order.

39.

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (0.9)^{2n}}}$$

$$n=0 \Rightarrow -3 \text{ dB}$$

$$n=1 \Rightarrow -2.577 \text{ dB}$$

$$n=3 \Rightarrow -1.851 \text{ dB}$$

$$n=5 \Rightarrow -1.299 \text{ dB}$$

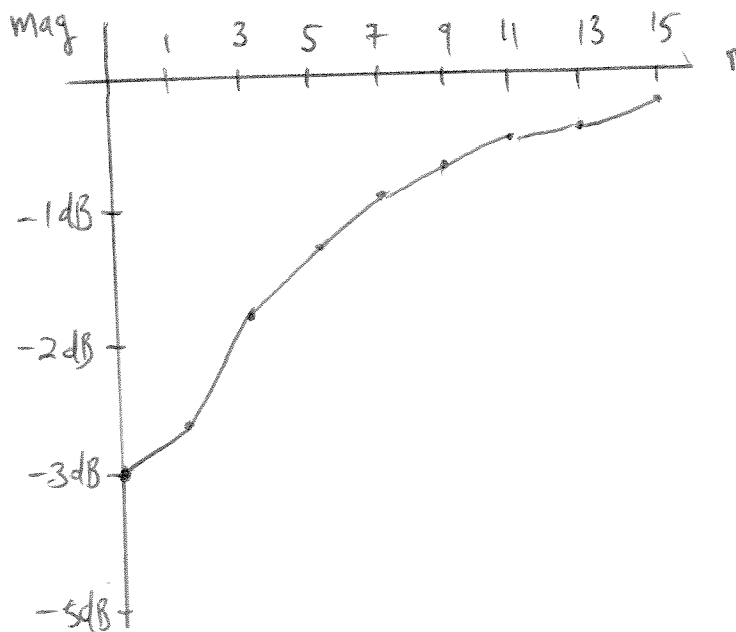
$$n=7 \Rightarrow -0.895 \text{ dB}$$

$$n=9 \Rightarrow -0.607 \text{ dB}$$

$$n=11 \Rightarrow -0.408 \text{ dB}$$

$$n=13 \Rightarrow -0.272 \text{ dB}$$

$$n=15 \Rightarrow -0.180 \text{ dB}$$



40.

$$1.1W_0$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (1.1)^{2n}}} = 0.1 \Rightarrow 2n = \frac{\log(99)}{\log(1.1)}$$

$n = 24.106$, so needs $n = 25$.

$$n=0 \Rightarrow -3 \text{ dB}$$

$$n=1 \Rightarrow -3.4439 \text{ dB}$$

$$n=3 \Rightarrow -4.427 \text{ dB}$$

$$n=5 \Rightarrow -5.555 \text{ dB}$$

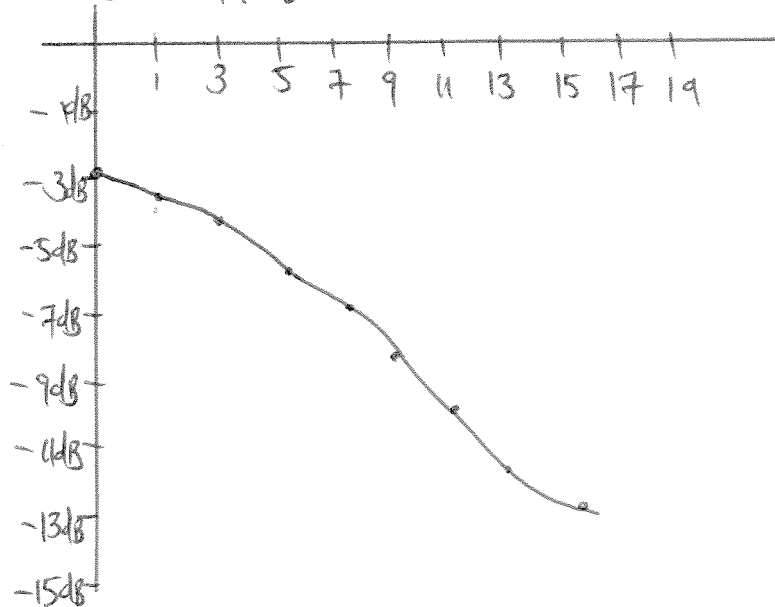
$$n=7 \Rightarrow -6.810 \text{ dB}$$

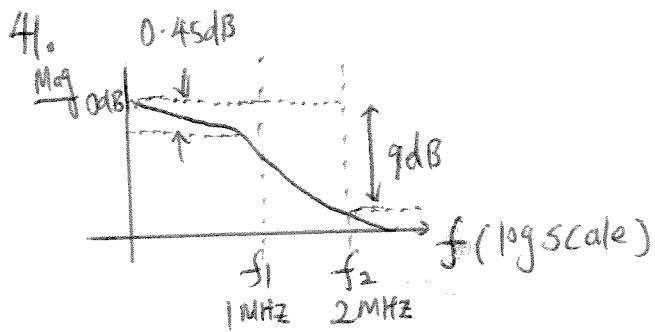
$$n=9 \Rightarrow -8.169 \text{ dB}$$

$$n=11 \Rightarrow -9.61 \text{ dB}$$

$$n=13 \Rightarrow -11.112 \text{ dB}$$

$$n=15 \Rightarrow -12.66 \text{ dB}$$





$$|H(f)| = \frac{1}{\left(1 + \left(\frac{2\pi f}{\omega_0}\right)^6\right)^{\frac{1}{2}}}$$

$$|H(5\text{MHz})| = 0.02438$$

$$\text{Suppression: } 20 \log(0.02438) = -32.26 \text{ dB}$$

42.

Low-pass Butterworth: Passband flatness of 0.5 dB
 $f_1 = 1 \text{ MHz}$, $f_2 = 2 \text{ MHz}$, order < 5

$$-0.5 \text{ dB} = 20 \log(x) \Rightarrow x = 0.944$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^{2n}}} = 0.944 \Rightarrow \frac{1}{\left(1 + \left(\frac{1}{f_0}\right)^{2n}\right)} = (0.944)^2$$

@ $f = 1 \text{ MHz}$

$$\Rightarrow 1 + \frac{1}{(f_0)^{2n}} = \frac{1}{(0.944)^2} \Rightarrow f_0 = 10^{\frac{0.91306}{2n}}$$

$$\text{for } n=1, f_0 \approx 2.86 \text{ MHz}$$

$$\text{for } n=5, f_0 \approx 1.234 \text{ MHz}$$

Therefore, for greatest attenuation $n=5$

$$\text{So } H(2 \text{ MHz}) = \frac{1}{\sqrt{1 + \left(\frac{2}{1.234}\right)^{10}}} = 0.089$$

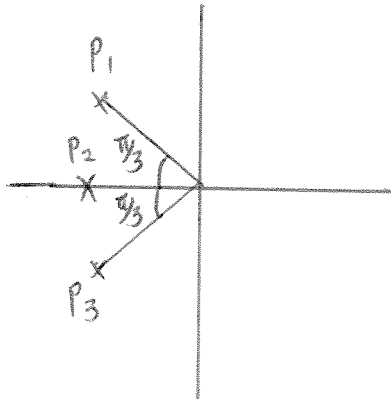
$$20 \log(0.089) = -21.0 \text{ dB at } n=5$$

43.

$$P_k = \omega_0 \exp\left(\frac{j\pi}{2}\right) \exp\left(j \frac{2k-1}{2n} \pi\right), \quad k=1, 2, \dots, n$$

The poles lie on a circle because all of their magnitude, which is the distance from the origin to the poles, are the same (ω_0) with each k ; only the phase, which is the angle the poles make with the positive real axis, differ. Therefore, a circle is formed.

44.



$$P_1 = 2\pi(1.45 \text{ MHz}) \left[\cos\left(\frac{2\pi}{3}\right) + j \sin\left(\frac{2\pi}{3}\right) \right]$$

$$P_2 = (2\pi)(1.45 \text{ MHz})$$

$$P_3 = (2\pi)(1.45 \text{ MHz}) \left[\cos\left(\frac{2\pi}{3}\right) - j \sin\left(\frac{2\pi}{3}\right) \right]$$

$$H(s) = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)} = \frac{[2\pi(1.45 \text{ MHz})]^2}{s^2 - [4\pi(1.45 \text{ MHz}) \cos(\frac{2\pi}{3})]s + [2\pi(1.45 \text{ MHz})]^2}$$

KHN Low pass Transfer function:

$$\frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2} \cdot \frac{1}{R_1 R_2 C_1 C_2 s^2} = \frac{\alpha / (R_1 R_2 C_1 C_2)}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

$$\frac{\alpha}{R_1 R_2 C_1 C_2} = (2\pi \times 1.45 \times 10^6)^2, \quad \omega_n^2 = \frac{R_6}{R_3} \left(\frac{1}{R_1 R_2 C_1 C_2} \right) = [2\pi \times 1.45 \times 10^6]^2$$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4 + R_5} \left(\frac{1}{R_1 C_1} \right) = -(4\pi \times 1.45 \times 10^6 \times \cos(\frac{2\pi}{3}))$$

$$\frac{\alpha}{R_1 R_2 C_1 C_2} = \frac{R_5}{R_4 + R_5} \left(1 + \frac{R_6}{R_3} \right) \left(\frac{1}{R_1 R_2 C_1 C_2} \right)$$

Let $R_6 = R_3$, $R_2 = 4R_1$, $C_1 = C_2$

$$\omega_n^2 = \left(\frac{1}{4R_1 C_1} \right)^2 = (2\pi \times 1.45 \times 10^6)^2 \Rightarrow \frac{1}{2R_1 C_1} = 2\pi \times 1.45 \times 10^6$$

44.

Let $R_1 = 5K \Rightarrow C_1 = 10.98pF$, $R_2 = 20K$, $C_2 = 10.98pF$

$$\frac{\omega_n}{Q} = \frac{R_4}{R_4 + R_5} \left(\frac{1}{R_1 C_1} \right) = 9110618.7$$

$$\Rightarrow \frac{R_4}{R_4 + R_5} = \frac{1}{2}$$

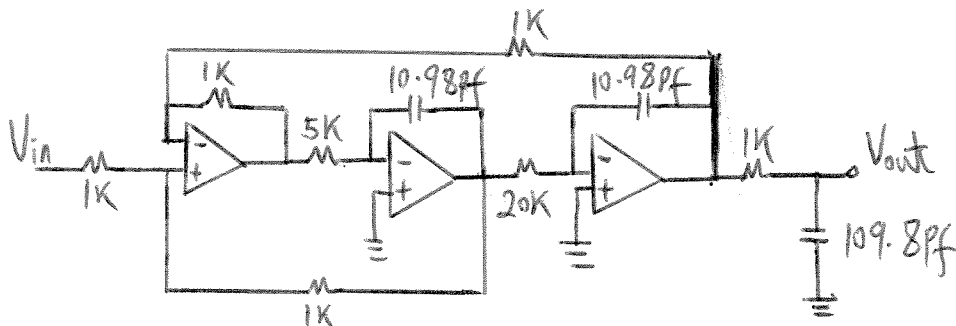
$$\frac{R_5}{R_4 + R_5} \left(1 + \frac{R_6}{R_3} \right) \left(\frac{1}{R_1 R_2 C_1 C_2} \right) = \left(\frac{1}{2} \right) (2) (2\pi \times 1.45 \times 10^6)^2 = (2\pi \times 1.45 \times 10^6)^2$$

Let R_5 and R_4 be $1K$ apiece.

So $R_5 = R_4 = R_6 = R_3 = 1K$

$R_1 = 5K$, $R_2 = 20K$

$C_1 = C_2 = 10.98pF$



45.

TOW-Thomas Biquad

$$\frac{V_Y}{V_{in}} = \frac{R_3 R_4}{R_1} \left(\frac{1}{R_2 R_3 R_4 C_1 C_2 s^2 + R_2 R_4 C_2 s + R_3} \right)$$

$$\frac{V_Y}{V_{in}} = \frac{1/(R_1 R_2 C_1 C_2)}{s^2 + 1/(R_3 C_1)s + 1/(R_2 R_4 C_1 C_2)}$$

$$\frac{V_Y}{V_{in}} = \frac{(2\pi \times 1.45 \times 10^6)^2}{s^2 - (4\pi \times 1.45 \times 10^6 \times \cos(2\pi/3))s + (2\pi \times 1.45 \times 10^6)^2}$$

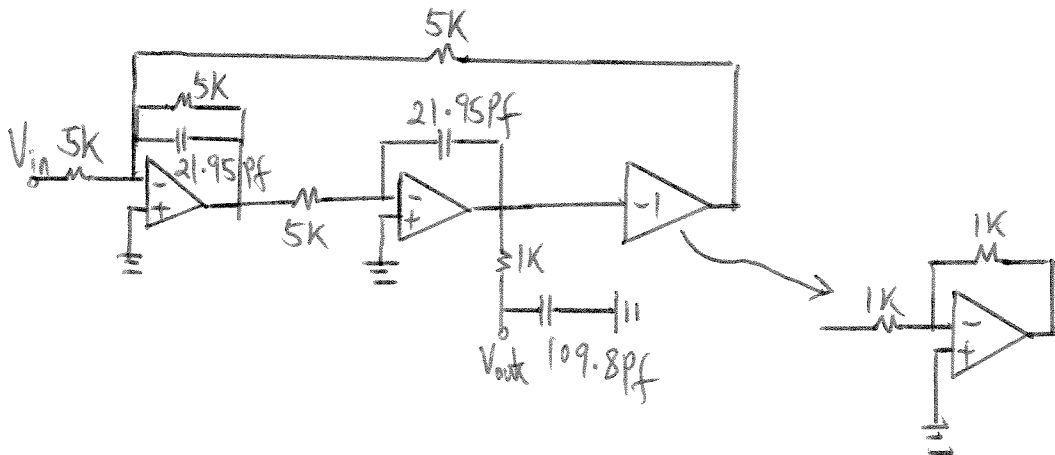
$$\frac{1}{R_1 R_2 C_1 C_2} = (2\pi \times 1.45 \times 10^6)^2, \frac{1}{R_2 R_4 C_1 C_2} = (2\pi \times 1.45 \times 10^6)^2$$

$$\frac{1}{R_3 C_1} = 2\pi \times 1.45 \times 10^6$$

Let $R_1 = R_2 = R_3 = R_4$, $C_1 = C_2$

let $R_3 = 5K \Rightarrow C_1 = 21.95 \text{ pf}$

So $R_1 = R_2 = R_3 = R_4 = 5k$, and $C_1 = C_2 = 21.95\mu f$



46.

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 C_n^2\left(\frac{\omega}{\omega_0}\right)}} \quad \begin{matrix} n=4 \\ \epsilon=0.2 \end{matrix}$$

$$C_n\left(\frac{\omega}{\omega_0}\right) = \cos\left(n \cos^{-1} \frac{\omega}{\omega_0}\right) = \cos\left(4 \cos^{-1} \frac{\omega}{\omega_0}\right)$$

$$C_n^2\left(\frac{\omega}{\omega_0}\right) = \cos^2\left(n \cos^{-1} \frac{\omega}{\omega_0}\right) = \frac{1}{2} (1 + \cos(2n \cos^{-1} \frac{\omega}{\omega_0}))$$

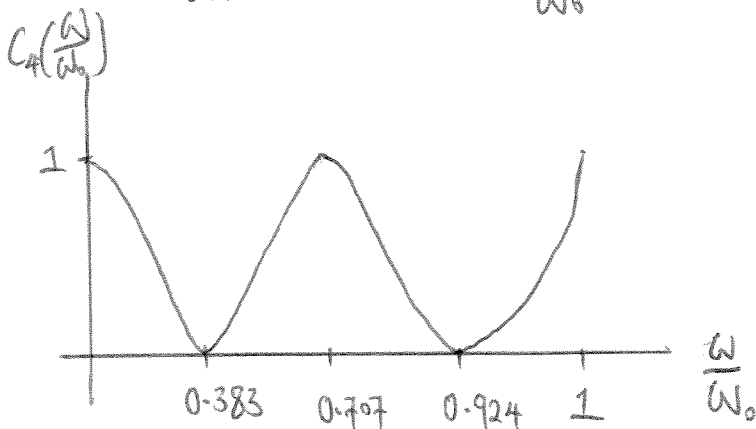
$$2n \cos^{-1} \frac{\omega}{\omega_0} = \pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.924$$

$$2n \cos^{-1} \frac{\omega}{\omega_0} = 3\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.383$$

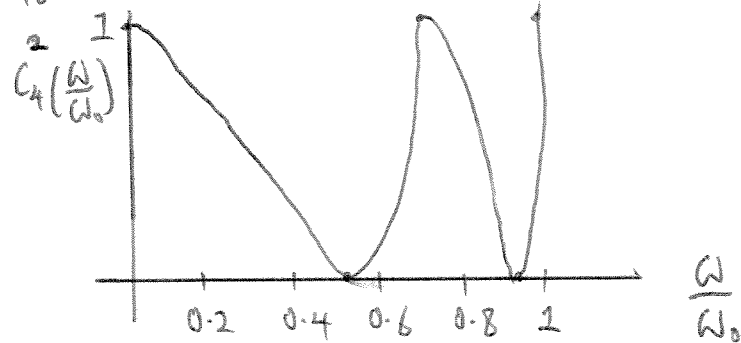
$$2n \cos^{-1} \frac{\omega}{\omega_0} = 0, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 1$$

$$2n \cos^{-1} \frac{\omega}{\omega_0} = 2\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0.707$$

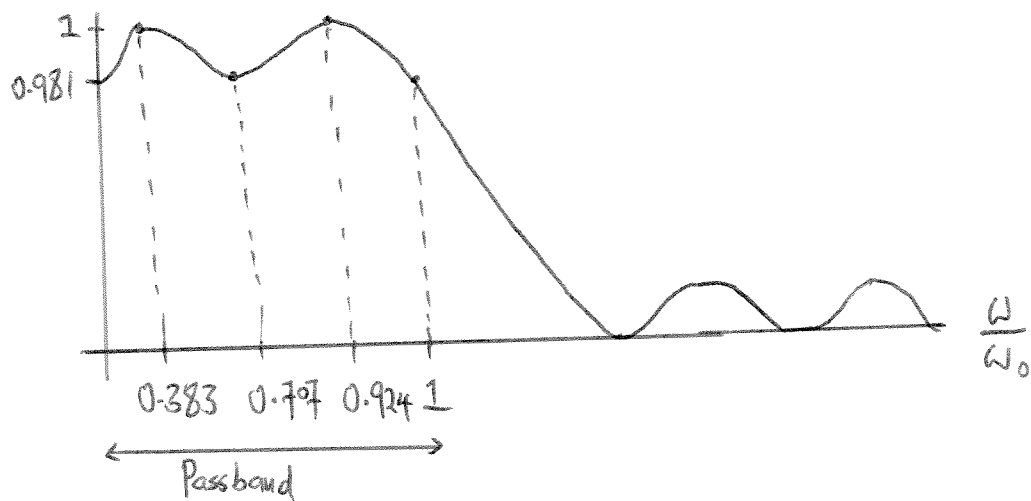
$$2n \cos^{-1} \frac{\omega}{\omega_0} = 4\pi, \quad n=4 \Rightarrow \frac{\omega}{\omega_0} = 0$$



46.



$$H(j\omega) = \frac{1}{\sqrt{1 + (0.2)^2 [C_4^2(\frac{\omega}{\omega_0})]}}$$



47. Chebyshev: 25 dB at 5 MHz.

$$n=5, \quad \omega_0 = 2\text{MHz}, \quad \frac{\omega}{\omega_0} = \frac{5}{2}.$$

$$\frac{1}{\sqrt{1 + \epsilon^2 \cosh^2(n \cosh^{-1} \frac{5}{2})}} = -25\text{dB} = 0.056234$$

$$\Rightarrow \frac{1}{1 + \epsilon^2 (1.5939 \times 10^6)} = 0.0031622771$$

$$\Rightarrow \epsilon^2 = 1.9777 \times 10^{-4}$$

$\Rightarrow$ Minimum Ripple

$$\frac{1}{\sqrt{1 + (1.9777 \times 10^{-4})}} = 0.99990 = -8.6 \times 10^{-4} \text{ dB.}$$

$$48. \quad n=6$$

$$\cosh^2 \left(6 \cos^{-1} \left(\frac{5}{2} \right) \right) = 36590401$$

$$\frac{1}{\sqrt{1 + \epsilon^2 (36590401)}} = 0.056234$$

$$\epsilon^2 = 8.615 \times 10^{-6}$$

$$\text{Minimum Ripple} = \frac{1}{\sqrt{1 + 8.615 \times 10^{-6}}} = -374 \times 10^{-5} \text{ dB}$$

Smaller than when $n=5$.

$$49. \quad \epsilon = 0.509, \quad n = 4$$

$$P_{1,4} = -0.140W_0 \pm 0.983jW_0$$

$$P_{2,3} = -0.337W_0 \pm 0.407jW_0$$

$$H_{1,4}(s) = \frac{0.986 W_0^2}{s^2 + 0.28 W_0 s + 0.986 W_0^2} = \frac{\alpha / (R_1 R_2 C_1 C_2)}{s^2 + \frac{W_n}{Q} s + W_n^2}$$

$$Q = 3.55$$

$$W_n = (2\pi)(4.965 \text{ MHz})$$

$$W_n^2 = [(2\pi)(4.965 \times 10^6)]^2 = \frac{R_6}{R_3} \left(\frac{1}{R_1 R_2 C_1 C_2} \right)$$

$$\frac{W_n}{Q} = (0.28)(5 \text{ MHz})(2\pi) = (1.4 \text{ MHz})(2\pi) = \frac{R_4}{R_4 + R_5} \left(\frac{1}{R_1 C_1} \right)$$

$$\frac{R_5}{R_4 + R_5} \left(1 + \frac{R_6}{R_3} \right) \left(\frac{1}{R_1 R_2 C_1 C_2} \right) = \frac{R_6}{R_3} \left(\frac{1}{R_1 R_2 C_1 C_2} \right)$$

$$\Rightarrow \frac{R_5}{R_4 + R_5} \left(1 + \frac{R_6}{R_3} \right) = \frac{R_6}{R_3}$$

$$\Rightarrow \frac{R_5}{R_4 + R_5} = \frac{\frac{R_6}{R_3}}{1 + \frac{R_6}{R_3}}$$

$$\Rightarrow 1 - \alpha = \frac{\frac{R_6}{R_3}}{1 + \frac{R_6}{R_3}}, \quad \alpha = \frac{R_4}{R_4 + R_5}$$

$$\Rightarrow \frac{R_6}{R_3} = \frac{1 - \alpha}{\alpha}$$

$$\text{Let } \alpha = 0.5 \Rightarrow \frac{R_6}{R_3} = 1$$

49.

$$\Rightarrow \omega_n^2 = \frac{R_6}{R_3} \left(\frac{1}{R_1 C_1 R_2 C_2} \right) = \frac{1}{R_1 C_1 R_2 C_2} \quad (*)$$

$$\text{Since } \frac{\omega_n}{Q} = \alpha \left(\frac{1}{R_1 C_1} \right) = 1.4 \times 10^6 \times 2\pi, \quad \alpha = 0.5$$

$$\Rightarrow \frac{1}{R_1 C_1} = 1.76 \times 10^7 \quad (1)$$

Consider (*)

$$\Rightarrow \frac{1}{R_2 C_2} = \omega_n^2 \cdot R_1 C_2 = 5.53 \times 10^7 \quad (2)$$

$R_6 = R_3 = R_5 = R_4 = 1k$. According to (1), (2), choose

$$R_1 = 5k, \quad C_1 = 11.368p$$

$$R_2 = 5k, \quad C_2 = 3.62p$$

$$\text{For } H_{2,3}(s) = \frac{0.279\omega_0^2}{s^2 + 0.674\omega_0 s + 0.279\omega_0^2}$$

$$\omega_n = (2\pi) (2.64 \times 10^6)$$

$$\frac{\omega_n}{Q} = (2\pi) (0.674 \times 5 \times 10^6) = (2\pi) (3.37 \times 10^6)$$

$$\text{Let } \alpha = 0.5$$

$$49. \frac{W_n}{Q} = (\alpha) \left(\frac{1}{R_1 C_1} \right)$$

$$\Rightarrow \frac{1}{R_1 C_1} = \frac{W_n}{Q} \cdot \frac{1}{\alpha} = 4.23 \times 10^7 \quad (3)$$

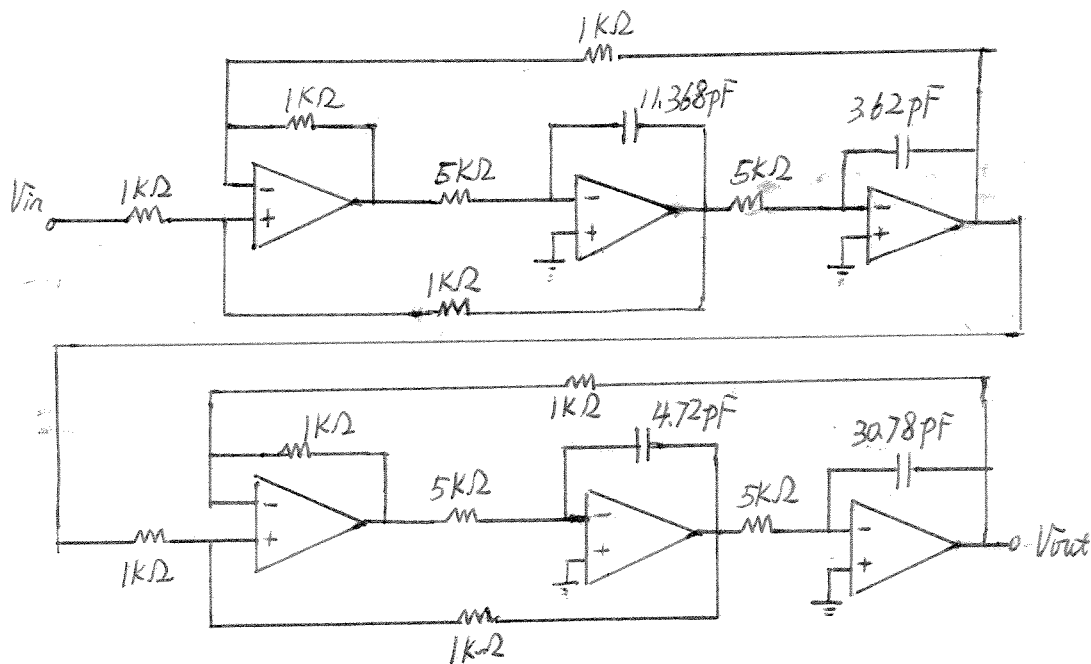
$$\frac{R_6}{R_3} = 1, \quad W_n^2 = \frac{R_6}{R_3} \left(\frac{1}{R_1 C_1 R_2 C_2} \right) = \frac{1}{R_1 C_1 R_2 C_2}$$

$$\Rightarrow \frac{1}{R_2 C_2} = W_n^2 \cdot R_1 C_1 = 6.50 \times 10^6 \quad (4)$$

Consider (3) (4), choose

$$R_1 = 5K, \quad C_1 = 4.72p; \quad R_2 = 5K, \quad C_2 = 30.78p$$

$$R_6 = R_3 = R_5 = R_4 = 1K.$$



50. Tow Thomas

Low Pass Transfer Function

$$P_{1,4} = \frac{V_4}{V_{in}} = \frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + \frac{1}{R_3 C_1} s + \frac{1}{R_2 R_4 C_1 C_2}} = \frac{0.986 \omega_0^2}{s^2 + 0.28 \omega_0 s + 0.986 \omega_0^2}$$

$$\frac{1}{R_1 R_2 C_1 C_2} = \frac{1}{R_2 R_4 C_1 C_2} = [(12\pi)(4.965 \times 10^6)]^2 \quad (*)$$

$$\frac{1}{R_3 C_1} = 2\pi \times 1.4 \times 10^6$$

$$\text{Let } R_3 = 5K, \quad C_1 = 22.736p$$

$$\text{Let } C_1 = C_2, \quad C_2 = 22.736p$$

$$\begin{aligned} R_1 = R_2 &\stackrel{+}{\Rightarrow} R_1 = R_2 = R_4 = 1.4K \\ R_3 = 5K, \quad C_1 = C_2 = 22.736p &\quad \left. \vphantom{\begin{aligned} R_1 = R_2 \\ R_3 = 5K, \quad C_1 = C_2 = 22.736p \end{aligned}} \right\} \text{ for } P_{1,4} \end{aligned}$$

$$P_{2,3} = \frac{0.279 \omega_0^2}{s^2 + 0.674 \omega_0 s + 0.279 \omega_0^2} = \frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + \frac{1}{R_3 C_1} s + \frac{1}{R_2 R_4 C_1 C_2}}$$

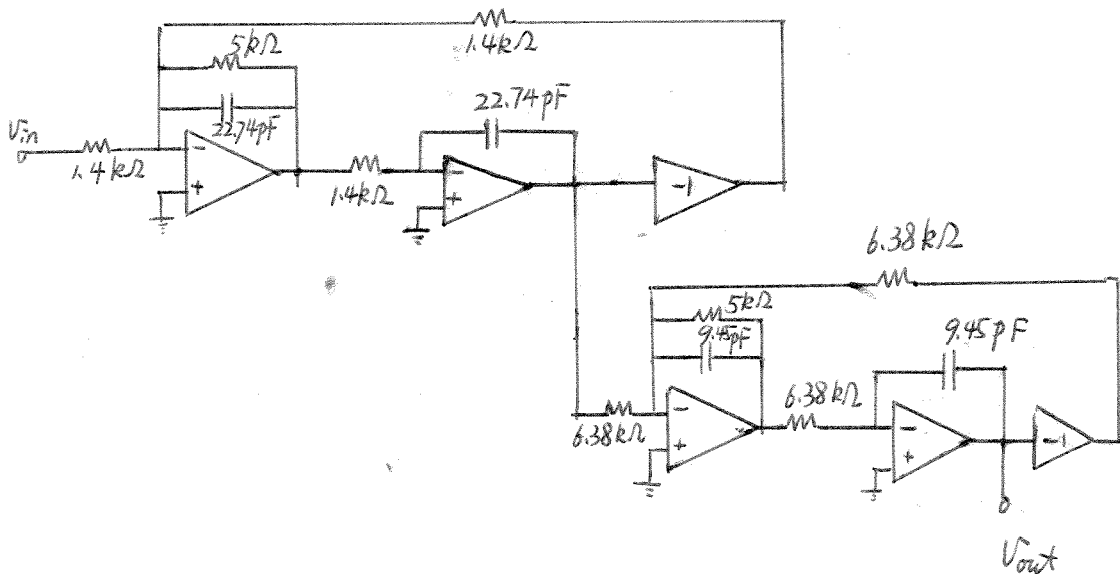
$$\frac{1}{R_1 R_2 C_1 C_2} = \frac{1}{R_2 R_4 C_1 C_2} = [(12\pi)(2.64 \times 10^6)]^2 \quad (*)$$

$$\frac{1}{R_3 C_1} = (12\pi)(3.37 \times 10^6)$$

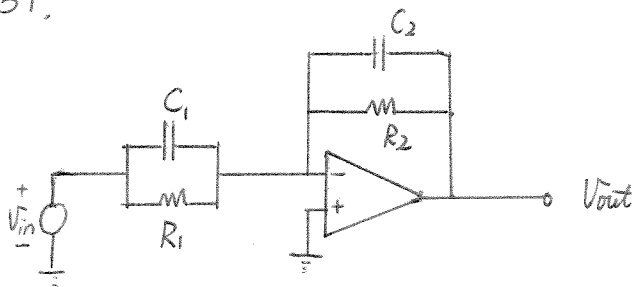
50. Let $R_3 = 5K$, $C_1 = 9.45 p$

Let $C_1 = C_2 = 9.45 p$, $R_1 = R_2 \xRightarrow{(*)'} R_1 = R_2 = R_4 = 6.38K$.

For $P_{1/4}$: $\begin{cases} R_1 = R_2 = R_4 = 6.38K, R_3 = 5K \\ C_1 = C_2 = 9.45 p. \end{cases}$

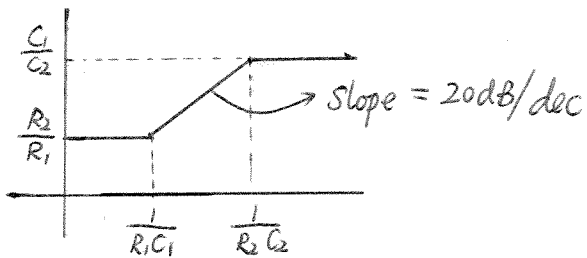


51.



High pass, $1\text{MHz} \Rightarrow 10\text{dB atten}$

$f > 5\text{MHz}$, $\text{gain} = 1$



$$\frac{1}{R_2 C_2} = (5\text{MHz})(2\pi)$$

$$\text{Let } \frac{C_1}{C_2} = 1, \quad \frac{R_2}{R_1} = -10\text{dB} = 0.316$$

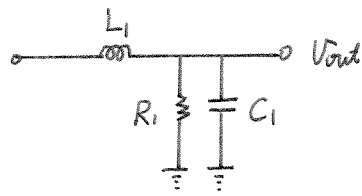
$$\text{So } \frac{1}{0.316} = 3.1623, \quad \frac{5\text{MHz}}{3.1623} = 1.58\text{MHz}$$

$$\Rightarrow \frac{1}{R_1 C_1} = (1.58\text{MHz})(2\pi)$$

$$\text{Choose } C_2 = 31.83\text{pF} \Rightarrow R_2 = 1\text{k}\Omega$$

$$C_1 = 31.83\text{pF} \Rightarrow R_1 = 3.16\text{k}\Omega$$

52.



Peaking : 1dB

bandwidth: 100 MHz

 $L_1 < 100 \text{ nH}$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{L_1 C_1}}{s^2 + \frac{1}{R_1 C_1} s + \frac{1}{L_1 C_1}} = \frac{\gamma}{s^2 + \frac{W_n}{Q} s + W_n^2} \bigg|_{s=j\omega} = \frac{\gamma}{(j\omega)^2 + \frac{W_n}{Q} (j\omega) + W_n^2}$$

$$H(j\omega) = \frac{\gamma}{(W_n^2 - \omega^2) + \frac{W_n}{Q} \omega j}$$

$$|H(j\omega)| = \frac{\gamma}{\sqrt{(W_n^2 - \omega^2)^2 + \left(\frac{W_n}{Q} \omega\right)^2}}$$

$$\text{At } \omega_1, |H(j\omega_1)| = \frac{\gamma}{\sqrt{(W_n^2 - \omega_1^2)^2 + \left(\frac{W_n}{Q} \omega_1\right)^2}} = \frac{\gamma}{W_n^2 \sqrt{2}}$$

$$\Rightarrow \sqrt{\frac{(W_n^2 - \omega_1^2)^2 + \left(\frac{W_n}{Q} \omega_1\right)^2}{W_n^4}} = \sqrt{2}$$

$$\Rightarrow (W_n^2 - \omega_1^2)^2 + \left(\frac{W_n}{Q} \omega_1\right)^2 = 2W_n^4 \quad (*)$$

$$\frac{Q}{\sqrt{1 - (4Q^2)^{-1}}} = 1.1 \Rightarrow Q = 0.9258, 0.5941 (< \frac{1}{\sqrt{2}}, \text{ can't produce peaking})$$

So $Q = 0.9258$.Solve (*) gives $\omega = \sqrt{1.5} W_n$.

$$\omega = \sqrt{1.5} \frac{1}{\sqrt{L_1 C_1}} = 127 (100 \times 10^6) \quad (1)$$

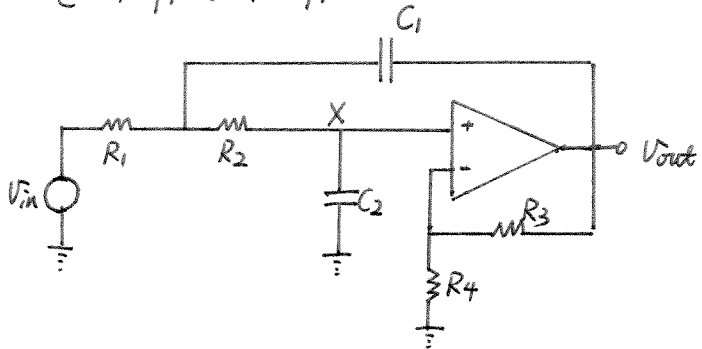
52.

$$\frac{W_n}{Q} = \frac{1}{R_1 C_1} \Rightarrow Q = R_1 C_1 \frac{1}{\sqrt{4C_1}} = R_1 \sqrt{\frac{C_1}{L_1}} = 0.9258 \quad (2)$$

$$\text{Let } L_1 = 90 \text{ nH} \xRightarrow{(1)} C_1 = 42.22 \text{ pF} \xRightarrow{(2)} R_1 = 42.74 \Omega.$$

53. $\omega_n = (2\pi)(50\text{MHz})$, $Q = 1.5$, Low frequency gain = 2.

$C = 10\text{pF}$ to 100pF .



$$\frac{V_{out}}{V_{in}} = \frac{1 + \frac{R_3}{R_4}}{R_1 R_2 C_1 C_2 S^2 + (R_1 C_2 + R_2 C_2 - R_1 \frac{R_3}{R_4} C_1) S + 1}$$

$$= \frac{(1 + \frac{R_3}{R_4}) / (R_1 R_2 C_1 C_2)}{S^2 + \frac{(R_1 C_2 + R_2 C_2 - \frac{R_1 R_3}{R_4} C_1) S + \frac{1}{R_1 R_2 C_1 C_2}}{R_1 R_2 C_1 C_2}}$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad \frac{\omega_n}{Q} = \frac{R_1 C_2 + R_2 C_2 - R_1 \frac{R_3}{R_4} C_1}{R_1 R_2 C_1 C_2}$$

Low frequency gain $(1 + \frac{R_3}{R_4}) = 2$, Let $\frac{R_3}{R_4} = 1$.

$$\frac{1}{\sqrt{R_1 R_2 C_1 C_2}} = \omega_n = (2\pi)(50 \times 10^6) \quad (1)$$

$$R_1 C_2 + R_2 C_2 - R_1 C_1 = \frac{\omega_n}{Q} (R_1 R_2 C_1 C_2) = \frac{1}{(1.5)(2\pi)(50 \times 10^6)} \quad (2)$$

Let $C_1 = C_2 = 10\text{pF}$

$$(2) \Rightarrow R_2 = \frac{1}{(1.5)(2\pi)(50 \times 10^6)} \cdot \frac{1}{C_2} = 212.2 (\Omega)$$

$$(1) \Rightarrow R_1 = \sqrt{[(2\pi)^2 (50 \times 10^6)^2 \cdot R_2 C_1 C_2]} = 477.5 (\Omega)$$

Let $R_3 = R_4 = 1\text{k}\Omega$.

54. $W_{3dB} = (30 \times 10^6) (2Z)$, gain = 2, sensitivities no greater than 1.

$$H(s) = \frac{K W_n^2}{s^2 + \frac{W_n}{Q} s + W_n^2}, \quad s = j\omega \Rightarrow$$

$$H(j\omega) = \frac{K W_n^2}{W_n^2 - \omega^2 + \frac{W_n}{Q} j\omega}$$

$$|H(j\omega)| = \frac{K W_n^2}{\sqrt{(W_n^2 - \omega^2)^2 + \left(\frac{W_n}{Q} \omega\right)^2}}$$

$$|H(j\omega)| = \frac{K}{\sqrt{2}} \Rightarrow \frac{W_n^2}{\sqrt{(W_n^2 - \omega^2)^2 + \left(\frac{W_n}{Q} \omega\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow (W_n^2 - \omega^2)^2 + \left(\frac{W_n}{Q} \omega\right)^2 = 2W_n^4$$

$$\Rightarrow W_n^4 \left(1 - \frac{\omega^2}{W_n^2}\right)^2 + W_n^4 \left[\left(\frac{1}{Q}\right)^2 \cdot \left(\frac{\omega}{W_n}\right)^2\right] = 2W_n^4$$

$$\Rightarrow \left[1 - \left(\frac{\omega}{W_n}\right)^2\right]^2 + \left(\frac{1}{Q}\right)^2 \left(\frac{\omega}{W_n}\right)^2 = 2$$

$$\Rightarrow \left(\frac{\omega}{W_n}\right)^4 + \left[\left(\frac{1}{Q}\right)^2 - 2\right] \left(\frac{\omega}{W_n}\right)^2 - 1 = 0$$

$$S_{R_2, C_1, C_2, R_1}^{W_n} = -\frac{1}{2} \quad (\text{sensitivities of } W_n \text{ all } < 1)$$

$$S_{R_1}^Q = -S_{R_2}^Q = -\frac{1}{2} + Q \sqrt{\frac{R_2 C_2}{R_1 C_1}}$$

$$S_{C_1}^Q = -S_{C_2}^Q = -\frac{1}{2} + Q \left(\sqrt{\frac{R_1 C_2}{R_2 C_1}} + \sqrt{\frac{R_2 C_2}{R_1 C_1}} \right) = \frac{1}{2} + Q \sqrt{\frac{R_1 C_1}{R_2 C_2}}$$

$$S_K^Q = QK \sqrt{\frac{R_1 C_1}{R_2 C_2}} = 2Q \sqrt{\frac{R_1 C_1}{R_2 C_2}}$$

Let $\sqrt{\frac{R_1 C_2}{R_2 C_1}} = 1$, and $Q = \frac{1}{2}$,

$$S_K^Q = 2 \cdot \left(\frac{1}{2}\right) = 1, \quad S_{C_1}^Q = \frac{1}{2} + \frac{1}{2} = 1$$

$$S_{C_2}^Q = -1, \quad S_{R_1}^Q = -\frac{1}{2} + \frac{1}{2} = 0, \quad S_{R_2}^Q = 0$$

Since $Q = \frac{1}{2}$,

$$\left(\frac{W}{W_n}\right)^4 + 2 \left(\frac{W}{W_n}\right)^2 - 1 = 0$$

$$\Rightarrow \left(\frac{W}{W_n}\right)^2 = 0.4142$$

$$\Rightarrow W = \sqrt{0.4142} W_n.$$

Since $R_1 C_1 = R_2 C_2$,

$$W_n = \frac{1}{\sqrt{(R_1 C_1)^2}} = \frac{1}{R_1 C_1}$$

$$\Rightarrow \sqrt{0.4142} W_n = \frac{\sqrt{0.4142}}{R_1 C_1} = (2\pi)(30 \times 10^6) \quad (1)$$

$$\text{Also } \frac{1}{Q W_n} = R_1 C_2 + R_2 C_2 - R_1 C_1 = R_1 C_2 \quad (2)$$

$$\Rightarrow \frac{R_1 C_1}{R_1 C_2} = \frac{\frac{1}{W_n}}{\frac{1}{Q W_n}} = Q = \frac{1}{2}.$$

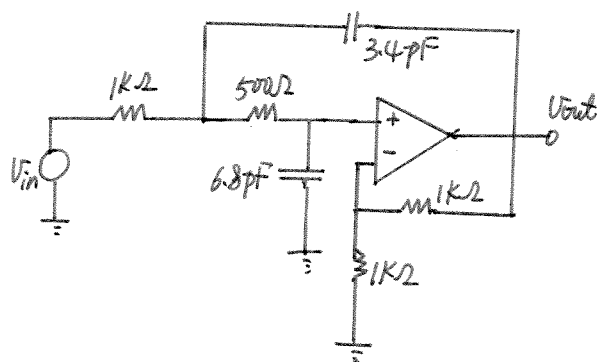
$$\Rightarrow \frac{C_1}{C_2} = \frac{1}{2}.$$

$$\text{Let } R_1 = 1K\Omega \xRightarrow{(1)} C_1 = \frac{\sqrt{0.4142}}{(2\pi)(30 \times 10^6) \cdot R_1} = 3.4pF.$$

$$\Rightarrow C_2 = 2C_1 = 6.8pF.$$

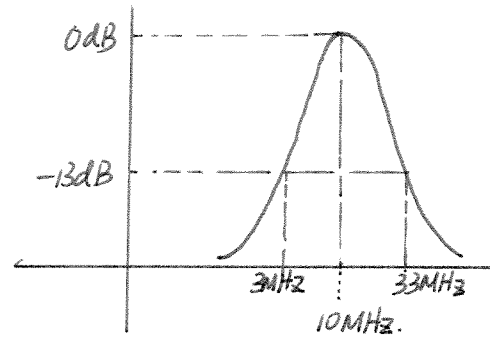
$$\Rightarrow R_2 = R_1 \frac{C_1}{C_2} = 500\Omega.$$

And as before, $R_3 = R_4 = 1K\Omega$.



55. 10MHz, Gain=1 (peak), $R_6=R_3$, -13dB @ 3MHz, 33MHz.

$$\frac{V_x}{V_{in}} = \frac{\alpha S^2}{S^2 + \frac{W_n}{Q} S + W_n^2} \cdot \frac{-1}{R_1 C_1 S}$$



$$1 = \left(\frac{\alpha}{R_1 C_1}\right) \cdot \frac{Q}{W_n}, \quad \alpha = \frac{R_5}{R_4 + R_5} \left(1 + \frac{R_6}{R_3}\right), \quad \frac{W_n}{Q} = \frac{R_4}{R_4 + R_5} \cdot \frac{1}{R_1 C_1}$$

$$\text{since } R_6 = R_3, \quad \alpha = 2 \frac{R_5}{R_4 + R_5} \Rightarrow \frac{\alpha}{2} = \frac{R_5}{R_4 + R_5}$$

$$\Rightarrow \frac{R_4}{R_4 + R_5} = 1 - \frac{\alpha}{2} \Rightarrow \frac{Q}{W_n} = \frac{R_1 C_1}{1 - \frac{\alpha}{2}}$$

$$\Rightarrow \left(\frac{\alpha}{R_1 C_1}\right) \cdot \left(\frac{R_1 C_1}{1 - \frac{\alpha}{2}}\right) = 1 \Rightarrow \alpha = 1 - \frac{\alpha}{2}$$

$$\Rightarrow \alpha = \frac{2}{3}, \quad \frac{R_5}{R_4 + R_5} = \frac{1}{3}$$

$$\Rightarrow R_5 = \frac{1}{2} R_4$$

$$\left. \begin{aligned} \frac{W_n}{Q} &= \left(\frac{2}{3}\right) \left(\frac{1}{R_1 C_1}\right) \\ W_n &= \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \end{aligned} \right\} \Rightarrow \frac{3}{2} \frac{R_1 C_1}{\sqrt{R_1 R_2 C_1 C_2}} = Q$$

$$\Rightarrow \frac{3}{2} \sqrt{\frac{R_1 C_1}{R_2 C_2}} = Q$$

$$\text{Let } R_1 C_1 = R_2 C_2 \Rightarrow Q = \frac{3}{2}, \quad W_n = \frac{1}{R_1 C_1}$$

$$H(j\omega) = \frac{\frac{2}{3}\omega^2}{\frac{\omega}{\omega_n} \sqrt{(\omega_n^2 - \omega^2)^2 + (\frac{2}{3}\omega_n\omega)^2}} = \frac{\frac{2}{3}\omega^2}{\frac{\omega}{\omega_n} \sqrt{\omega_n^4 - \frac{14}{9}(\omega_n\omega)^2 + \omega^4}}$$

$$H(j\omega) = 1$$

$$\Rightarrow \frac{4}{9}\omega^2 = \omega_n^2 - \frac{14}{9}\omega^2 + \frac{\omega^4}{\omega_n^2}$$

$$\Rightarrow \omega_n^4 - 2\omega^2\omega_n^2 + \omega^4 = 0$$

$$\Rightarrow \omega_n^2 = \omega^2 = [(2\pi)(10\text{MHz})]^2$$

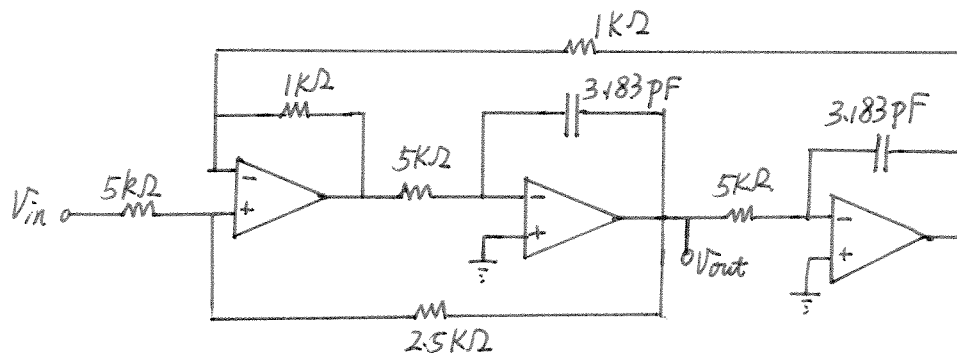
$$\text{As derived, } \omega_n = \frac{1}{R_1 C_1}$$

$$\text{Let } R_1 = 5\text{k}\Omega \Rightarrow C_1 = 3.183\text{pF.}$$

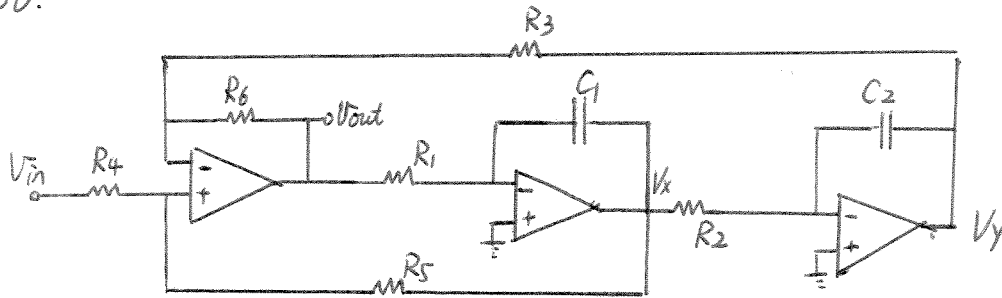
$$\text{Let } R_2 = R_1 = 5\text{k}\Omega \Rightarrow C_2 = C_1 = 3.183\text{pF}$$

$$\text{Let } R_4 = 5\text{k}\Omega \Rightarrow R_5 = \frac{1}{2}R_4 = 2.5\text{k}\Omega.$$

$$\text{Let } R_3 = R_6 = 1\text{k}\Omega.$$



56.



Low pass,
$$\frac{V_y}{V_{in}} = \frac{\alpha S^2}{S^2 + \frac{W_n}{Q} S + W_n^2} \cdot \frac{1}{R_1 R_2 C_1 C_2 S^2}$$

$$= \frac{\alpha}{(S^2 + \frac{W_n}{Q} S + W_n^2) (R_1 R_2 C_1 C_2)}$$

$$H(S) = \frac{\alpha / (R_1 R_2 C_1 C_2)}{S^2 + \frac{W_n}{Q} S + W_n^2}$$

$$H(jW) = \frac{\alpha / (R_1 R_2 C_1 C_2)}{(W_n^2 - W^2) + j \frac{W_n}{Q} W}$$

$$|H(jW)| = \frac{\alpha / (R_1 R_2 C_1 C_2)}{\sqrt{(W_n^2 - W^2)^2 + (\frac{W_n W}{Q})^2}}$$

$$|H(W_{3dB})| = \frac{\alpha / (R_1 R_2 C_1 C_2)}{\sqrt{(W_n^2 - W_{3dB}^2)^2 + (\frac{W_n W_{3dB}}{Q})^2}} = \frac{\alpha}{W_n^2 (R_1 R_2 C_1 C_2) \sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{(W_n^2 - W_{3dB}^2)^2 + (\frac{W_n W_{3dB}}{Q})^2}}{W_n^2} = \sqrt{2}$$

$$\Rightarrow 1 - 2 \left(\frac{W_{3dB}}{W_n} \right)^2 + \left(\frac{W_{3dB}}{W_n} \right)^4 + \frac{1}{Q^2} \left(\frac{W_{3dB}}{W_n} \right)^2 = 2$$

$$\Rightarrow \left(\frac{\omega_{3dB}}{\omega_n}\right)^4 + \left(\frac{1}{Q^2} - 2\right) \left(\frac{\omega_{3dB}}{\omega_n}\right)^2 - 1 = 0$$

$$Q = 1.5 \Rightarrow \left(\frac{\omega_{3dB}}{\omega_n}\right)^4 - 1.556 \left(\frac{\omega_{3dB}}{\omega_n}\right)^2 - 1 = 0$$

$$\Rightarrow \left(\frac{\omega_{3dB}}{\omega_n}\right)^2 = 2.0446, -0.4891 \text{ (impossible)}$$

$$\Rightarrow \omega_{3dB} = 1.43 \omega_n$$

$$\Rightarrow \text{Low pass corner} = 14.3 \text{ MHz.}$$

High pass:

$$\frac{V_{out}}{V_{in}}(s) = \frac{\alpha s^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

$$|H(j\omega)| = \frac{\alpha \omega^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + \left(\frac{\omega \omega_n}{Q}\right)^2}}$$

$$|H(j\omega_{3dB})| = \frac{\alpha \omega_{3dB}^2}{\sqrt{(\omega_n^2 - \omega_{3dB}^2)^2 + \left(\frac{\omega_{3dB} \omega_n}{Q}\right)^2}} = \frac{\alpha}{\sqrt{2}}$$

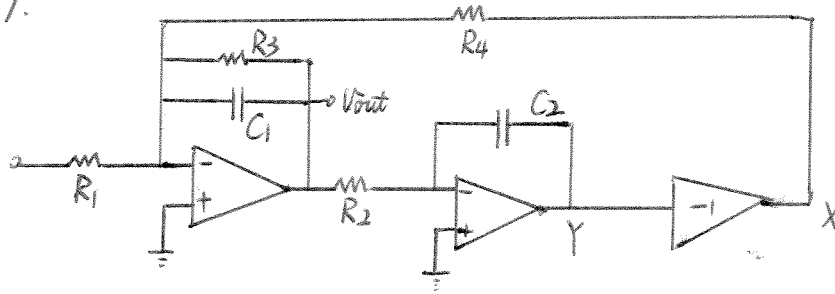
$$\Rightarrow \left(\frac{\omega_n}{\omega_{3dB}}\right)^4 + \left(\frac{1}{Q^2} - 2\right) \left(\frac{\omega_n}{\omega_{3dB}}\right)^2 - 1 = 0$$

Since $Q = 1.5$

$$\Rightarrow \left(\frac{\omega_n}{\omega_{3dB}}\right)^2 = 2.0446 \Rightarrow \omega_{3dB} = \frac{\omega_n}{1.43}$$

$$\Rightarrow \omega_{3dB} = 7 \text{ MHz. (high pass corner)}$$

57.



$$\omega_n = 10 \text{ MHz},$$

$$-13 \text{ dB} = 3 \text{ MHz},$$

$$33 \text{ MHz}.$$

$$\frac{V_{out}}{V_{in}} = - \frac{R_2 R_3 R_4}{R_1} \left(\frac{C_2 S}{R_2 R_3 R_4 C_1 C_2 S^2 + R_2 R_4 C_2 S + R_3} \right)$$

Same as in #55, $Q = \frac{10}{6.684} = 1.5.$

$$\frac{V_{out}}{V_{in}} = - \frac{\frac{1}{R_3 C_1} S}{S^2 + \frac{1}{R_3 C_1} S + \frac{1}{R_2 R_4 C_1 C_2}}$$

$$\frac{\omega_n}{Q} = \frac{1}{R_3 C_1}, \quad \omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}$$

$$Q = \frac{R_3 C_1}{\sqrt{R_2 R_4 C_1 C_2}} = R_3 \sqrt{\frac{C_1}{R_2 R_4 C_2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{-\beta S}{S^2 + \frac{\omega_n}{Q} S + \omega_n^2},$$

At $\omega = \omega_n \Rightarrow |H(j\omega_n)| = 1 = \frac{\beta Q}{\omega_n}.$

$$\frac{\beta Q}{\omega_n} = \left(\frac{1}{R_1 C_1} \right) (R_3 C_1) = \frac{R_3}{R_1} = 1$$

$$Q = R_3 \sqrt{\frac{C_1}{R_2 R_4 C_2}} = 15, \quad W_1 = \frac{1}{\sqrt{R_2 R_4 C_2 C_1}} = (10 \times 10^6)(12\pi)$$

$$\text{Let } R_2 = R_4 = 1 \text{ k}\Omega.$$

$$\frac{1}{\sqrt{10^6 \times C_1 C_2}} = (10 \times 10^6)(12\pi) \Rightarrow C_1 C_2 = 2.533 \times 10^{-22}$$

$$\text{Let } C_1 = C_2 = 15.9 \text{ pF}$$

$$R_3 \sqrt{\frac{1}{10000 \times 10000}} = 15.$$

$$\Rightarrow R_3 = 1.5 \text{ k}\Omega = R_1$$

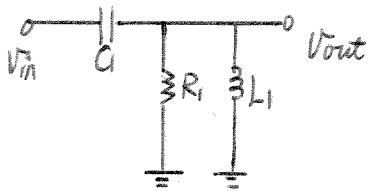
$$\text{So : } R_1 = 1.5 \text{ k}\Omega, R_2 = 1 \text{ k}\Omega, R_3 = 1.5 \text{ k}\Omega, R_4 = 1 \text{ k}\Omega.$$

$$C_1 = C_2 = 15.9 \text{ pF}.$$

58. Peaking : 1dB @ 7MHz.

Corner : 3.69MHz

-13.6dB @ 2MHz.



$$\frac{V_{out}}{V_{in}} = \frac{s^2}{s^2 + \frac{1}{R_1 C_1} s + \frac{1}{L_1 C_1}}$$

Peaking 1dB $\Rightarrow Q = 0.926$.

$$\frac{\omega_n}{\sqrt{1 - 1/(2Q^2)}} = (2\pi)(7\text{MHz}) \Rightarrow \omega_n = (2\pi)(4.52\text{MHz})$$

$$\frac{\omega_n}{Q} = \frac{(2\pi)(4.52\text{MHz})}{0.926} = (2\pi)(4.88\text{MHz}) = \frac{1}{R_1 C_1}$$

$$\omega_n^2 = [(2\pi)(4.52\text{MHz})]^2 = \frac{1}{L_1 C_1}$$

Let $C_1 = 100\text{pF}$, $L_1 = 12.4\mu\text{H}$, $R_1 = 326.1\Omega$.

With simulated inductor

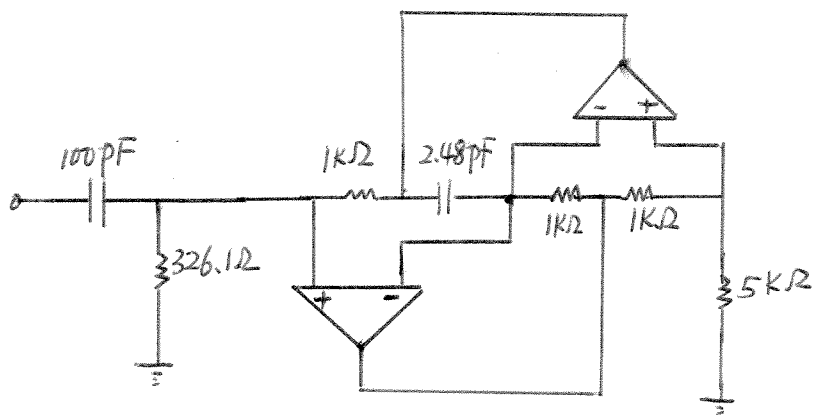
$$Z_{in} = \left(\frac{Z_1 Z_3}{Z_2 Z_4} \right) Z_5 = R_Y R_X C_S$$

Let $Z_1 = Z_3 = Z_4 = R_Y$, $Z_5 = R_X$, $Z_2 = C_S^{-1}$

Let $R_Y = 1\text{k}\Omega$, $R_X = 5\text{k}\Omega$.

$$12.4 \times 10^{-6} = (1000)(5000) C.$$

$\Rightarrow C = 2.48\text{pF}$ to simulate an L of $12.4\mu\text{H}$.



59. Corner @ 16.38 MHz, Peaking 0.5 dB @ 8 MHz.

5.9 dB $\approx$ 6 dB attenuation @ 20 MHz.

$$\frac{V_{out}}{V_{in}} = \frac{1}{R_1 R_x C^2 s^2 + R_1 C s + 1} = \frac{1/(R_1 R_x C^2)}{s^2 + \frac{s}{R_x C} + \frac{1}{R_1 R_x C^2}}$$

$$0.5 \text{ dB} \Leftrightarrow 1.05292.$$

$$\frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}} = 1.05292 \Rightarrow Q = 0.8636$$

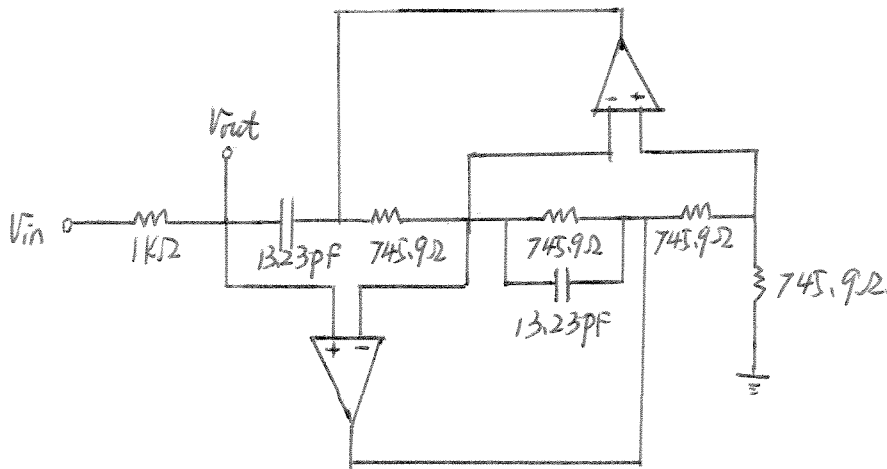
$$\omega_n \sqrt{1 - \frac{1}{2Q^2}} = (2\pi)(8 \times 10^6)$$

$$\Rightarrow \omega_n = (2\pi)(13.934 \times 10^6)$$

$$\frac{1}{R_1 R_x C^2} = \omega_n^2, \quad \frac{1}{R_x C} = \frac{\omega_n}{Q} = (2\pi)(16.134 \times 10^6)$$

$$\Rightarrow \frac{1}{R_1 C} = 7.56 \times 10^7$$

Let $R_1 = 1 \text{ k}\Omega$, $C = 13.23 \text{ pF}$, $R_x = 745.9 \Omega$.



60. Butterworth

a) Passband 0.5 dB @ 1 MHz, $-0.5 \text{ dB} \Leftrightarrow 0.944$

Attenuation 12 dB @ 2.5 MHz, $-12 \text{ dB} \Leftrightarrow 0.2512$

$$|H(j\omega)|_{1 \text{ MHz}}^2 = \frac{1}{1 + \left[\frac{(2\pi)(10^6)}{W_0} \right]^{2n}} = 0.944^2 \quad (1)$$

$$|H(j\omega)|_{2.5 \text{ MHz}}^2 = \frac{1}{1 + \left[\frac{2\pi \times 2.5 \times 10^6}{W_0} \right]^{2n}} = 0.2512^2 \quad (2)$$

$$(1) \Rightarrow 1 = (0.944)^2 \left[\left(\frac{2\pi \times 10^6}{W_0} \right)^{2n} + 1 \right]$$

$$\Rightarrow W_0^{2n} = 8.186 \times (2\pi \times 10^6)^{2n} \quad (3)$$

$$(2) \Rightarrow 1 = (0.2512)^2 \left[\frac{(2\pi \times 2.5 \times 10^6)^{2n}}{8.186 \times (2\pi \times 10^6)^{2n}} + 1 \right]$$

$$\Rightarrow n = 2.62$$

$$\text{So choose } n = 3. \quad (3) \Rightarrow W_0 = 2\pi \times 1.42 \text{ MHz}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{2\pi \times 1.42 \times 10^6} \right)^6}}$$

b). Passband : 0.1 dB @ 1 MHz

$$\frac{1}{1 + \left(\frac{2\pi \times 10^6}{W_0} \right)^{2n}} = (0.98855)^2 \quad (1)$$

Stopband attenuation : 12 dB @ 2.5 MHz

$$\frac{1}{1 + \left(\frac{2\pi \times 2.5 \times 10^6}{W_0} \right)^{2n}} = (0.2512)^2 \quad (2)$$

$$(1) \Rightarrow W_0^{2n} = 42.931 \times (2\pi \times 10^6)^{2n}$$

$$(2) \Rightarrow n = 3.52$$

$$\text{Choose } n = 4 \quad (3) \Rightarrow W_0 = 2\pi \times 1.6 \text{ MHz.}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{2\pi \times 1.6 \times 10^6} \right)^8}}$$

$$c). \text{ Passband } 1 \text{ dB @ } 1 \text{ MHz} \Rightarrow \frac{1}{1 + \left(\frac{2\pi \times 10^6}{W_0} \right)^{2n}} = (0.90)^2 \quad (4)$$

$$\text{Attenuation } 18 \text{ dB @ } 2.5 \text{ MHz} \Rightarrow \frac{1}{1 + \left(\frac{2\pi \times 2.5 \times 10^6}{W_0} \right)^{2n}} = (0.259)^2 \quad (5)$$

$$(4) \Rightarrow W_0^{2n} = 4.263 \times (2\pi \times 10^6)^{2n} \quad (6)$$

$$(5) \Rightarrow n = 3.0$$

$$\text{Choose } n = 3 \quad (6) \Rightarrow W_0 = 2\pi \times 1.27 \text{ MHz}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{2\pi \times 1.27 \times 10^6} \right)^6}}$$

$$d) \text{ Passband: } 0.5\text{dB @ } 1\text{MHz} \Rightarrow \frac{1}{1 + \left(\frac{2\pi \times 10^6}{W_0}\right)^{2n}} = 0.944^2 \quad (1)$$

$$\text{Attenuation: } 18\text{dB @ } 2.5\text{MHz} \Rightarrow \frac{1}{1 + \left(\frac{2\pi \times 2.5 \times 10^6}{W_0}\right)^{2n}} = 0.1259^2 \quad (2)$$

$$(1) \Rightarrow W_0^{2n} = 8.186 \times (2\pi \times 10^6)^{2n} \quad (3)$$

$$(2) \Rightarrow n = 3.4$$

$$\text{Choose } n = 4 \xRightarrow{(3)} W_0 = 2\pi \times 1.3\text{MHz}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{2\pi \times 1.3 \times 10^6}\right)^8}}$$

Chebyshev

$$a) \text{ Passband } 0.5\text{dB @ } 1\text{MHz} \Rightarrow 0.5 = 20 \log(\sqrt{1 + \epsilon^2})$$

$$\Rightarrow \epsilon = 0.3493, W_0 = 1\text{MHz}$$

$$\text{Attenuation } 12\text{dB @ } 2.5\text{MHz}$$

$$\Rightarrow \frac{1}{\sqrt{1 + \epsilon^2 \cosh^2[n \cosh^{-1}(\frac{\omega}{W_0})]}} = 0.2512, \text{ when } \omega = 2.5 \times 10^6 \times 2\pi$$

Since ω, W_0, ϵ , known

$$\Rightarrow n = 1.9733$$

$$\text{Choose } n = 2, |H(j\omega)| = \frac{1}{\sqrt{1 + 0.3493^2 \cosh^2\left(2 \cosh^{-1}\left(\frac{\omega}{W_0}\right)\right)}}, W_0 = 2\pi \times 1\text{MHz}$$

b). Passband 0.1 dB @ 1 MHz, $\omega_0 = 1 \text{ MHz}$

$$\Rightarrow 0.1 = 20 \log(\sqrt{1+\epsilon^2}) \Rightarrow \epsilon = 0.1526.$$

Attenuation 12 dB @ 25 MHz

$$\Rightarrow \frac{1}{1 + 0.1526^2 \cosh^2[n \cosh^{-1}(25)]} = 0.2512^2$$

$$\Rightarrow n = 2.5$$

$$\text{Choose } n=3, |H(j\omega)| = \frac{1}{\sqrt{1 + 0.1526^2 \cosh^2\left(3 \cosh^{-1}\left(\frac{\omega}{\omega_0}\right)\right)}}, \omega_0 = (2\pi)(1 \text{ MHz})$$

c). Passband 1 dB @ 1 MHz, $\omega_0 = 1 \text{ MHz}$

$$\Rightarrow 1 = 20 \log \sqrt{1+\epsilon^2} \Rightarrow \epsilon = 0.5089.$$

Attenuation 18 dB @ 25 MHz

$$\Rightarrow \frac{1}{1 + 0.5089^2 \cosh^2[n \cosh^{-1}(25)]} = 0.1259^2 \Rightarrow n = 2.19$$

$$\text{Choose } n=3, |H(j\omega)| = \frac{1}{\sqrt{1 + 0.5089^2 \cosh^2\left(3 \cosh^{-1}\left(\frac{\omega}{\omega_0}\right)\right)}}, \omega_0 = (2\pi)(1 \text{ MHz})$$

d). Passband 0.5 dB @ 1 MHz $\Rightarrow \epsilon = 0.3493$

Attenuation 18 dB @ 25 MHz

$$\Rightarrow \frac{1}{1 + 0.3493^2 \cosh^2[n \cosh^{-1}(25)]} = 0.1259^2 \Rightarrow n = 2.43$$

$$\text{Choose } n=3, |H(j\omega)| = \frac{1}{\sqrt{1 + 0.3493^2 \cosh^2\left(3 \cosh^{-1}\left(\frac{\omega}{\omega_0}\right)\right)}}, \omega_0 = (2\pi)(1 \text{ MHz})$$

61. a) Butterworth in Sallen and Key

$$\omega_0 = 2\pi \times 1.42 \text{ MHz}$$

$$P_k = \omega_0 \cdot \exp\left(j\frac{\pi}{2}\right) \exp\left(j\frac{2k-1}{2n}\pi\right), \quad k=1, 2, 3.$$

$$P_1 = \omega_0 \exp\left(j\frac{2\pi}{3}\right) = (2\pi)(1.42 \text{ MHz}) \times \left(\cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}\right).$$

$$P_2 = \omega_0 \exp(j\pi) = -(2\pi)(1.42 \text{ MHz})$$

$$P_3 = \omega_0 \exp\left(j\frac{4\pi}{3}\right) = (2\pi)(1.42 \text{ MHz}) \times \left(\cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3}\right)$$

$$H_{p,3}(s) = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)}$$

$$= \frac{[2\pi \times (1.42 \text{ MHz})]^2}{s^2 - [4\pi \times (1.42 \text{ MHz}) \cos\frac{2\pi}{3}]s + [2\pi \times (1.42 \text{ MHz})]^2}$$

$$\omega_n = 2\pi \times 1.42 \text{ MHz} \left(= \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \right)$$

$$\frac{\omega_n}{Q} = 2\pi \times 1.42 \text{ MHz} \cdot \cos\left(\frac{2\pi}{3}\right) \Rightarrow Q = \frac{-1}{2 \cos\left(\frac{2\pi}{3}\right)} \left(= \frac{1}{R_1 + R_2} \sqrt{R_1 R_2 \frac{C_1}{C_2}} \right)$$

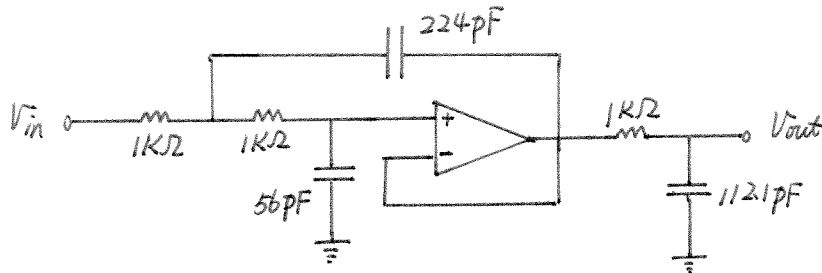
$$\text{Let } C_1 = 4C_2, R_1 = R_2, \text{ so that it satisfies } Q = \frac{-1}{2 \cos\left(\frac{2\pi}{3}\right)} = 1$$

$$\text{Also } \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} = 2\pi \times 1.42 \text{ MHz}$$

$$\text{Let } R_1 = R_2 = 1 \text{ k}\Omega \Rightarrow C_1 = 224 \text{ pF}, C_2 = 56 \text{ pF}$$

$$P_2 = -\omega_0, \quad \frac{1}{R_3 C_3} = (2\pi)(1.42 \text{ MHz})$$

$$\text{Let } R_3 = 1 \text{ k}\Omega \Rightarrow C_3 = 112.1 \text{ pF}$$



Chebyshev in Sallen and Key

$$P_k = -\omega_0 \sin \frac{(2k-1)\pi}{2n} \sinh \left(\frac{1}{n} \sinh^{-1} \frac{1}{\epsilon} \right) + j\omega_0 \cos \frac{(2k-1)\pi}{2n} \cosh \left(\frac{1}{n} \sinh^{-1} \frac{1}{\epsilon} \right)$$

$$n=2, \quad \omega_0 = (2\pi)(1 \text{ MHz}), \quad \epsilon = 0.3493, \quad K=1, 2.$$

$$P_{1,2} = -0.7128\omega_0 \pm j1.0041\omega_0$$

$$H_{SK}(s) = \frac{(s-P_1)(s-P_2)}{(s-P_1)(s-P_2)} = \frac{(1.2314)^2 \omega_0^2}{s^2 + 1.4256\omega_0 s + (1.2314)^2 \omega_0^2}$$

$$\omega_n = 1.2314\omega_0 = (2\pi)(1.2314 \text{ MHz})$$

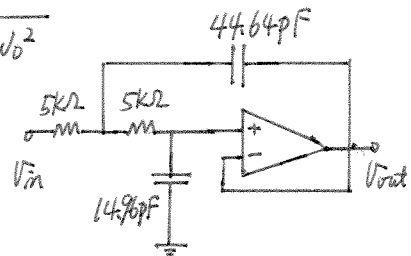
$$\frac{\omega_n}{Q} = 1.4256\omega_0 \Rightarrow Q = \frac{1.2314}{1.4256} = 0.8638$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad Q = \frac{1}{R_1 + R_2} \sqrt{R_1 R_2 \frac{C_1}{C_2}}$$

$$\text{Let } R_1 = R_2 \quad Q = \frac{1}{2} \sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 2.9844$$

$$\frac{1}{\sqrt{R_1 R_2 C_1 C_2}} = \omega_n = 2\pi(1.2314 \text{ MHz}), \Rightarrow \frac{1}{R_1 C_2 \sqrt{2.9844}} = 2\pi(1.2314 \text{ MHz})$$

$$\text{Let } R_1 = R_2 = 5 \text{ k}\Omega \Rightarrow C_2 = 14.96 \text{ pF}, \quad C_1 = 44.64 \text{ pF}$$



b). Butterworth with SK

$$n=4, \omega_0 = (2\pi)(1.6\text{MHz})$$

$$P_k = \omega_0 \exp(j\frac{\pi}{2}) \exp(j\frac{2k-1}{2n}\pi), \quad k=1, 2, 3, 4.$$

$$P_1 = \omega_0 \exp(j\frac{5\pi}{8}), \quad P_2 = \omega_0 \exp(j\frac{7\pi}{8}), \quad P_3 = \omega_0 \exp(-j\frac{5\pi}{8}), \quad P_4 = \omega_0 \exp(-j\frac{7\pi}{8})$$

$$H_{SK1,4}(s) = \frac{(-P_1)(-P_4)}{(s-P_1)(s-P_4)} = \frac{[(2\pi)(1.6 \times 10^6)]^2}{s^2 - [4\pi(1.6 \times 10^6) \cos(\frac{5\pi}{8})]s + [2\pi(1.6 \times 10^6)]^2}$$

$$\omega_n = 2\pi \times 1.6 \times 10^6$$

$$\frac{\omega_n}{Q} = (4\pi)(1.6 \times 10^6) \cos(\frac{5\pi}{8}) \Rightarrow Q = 1.31.$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad Q = \frac{1}{R_1 + R_2} \sqrt{R_1 R_2 \frac{C_1}{C_2}}.$$

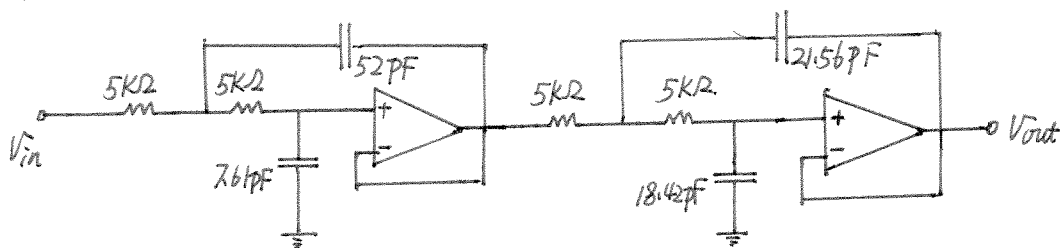
$$\text{Let } R_1 = R_2, \quad Q = \frac{1}{2} \sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 6.83.$$

$$\omega_n = \frac{1}{\sqrt{6.83} R_1 C_2} = 2\pi \times 1.6 \times 10^6$$

$$\text{Let } R_1 = R_2 = 5\text{K}\Omega \Rightarrow C_2 = 761\text{pF}, \quad C_1 = 52\text{pF}.$$

$$\text{Similarly, } H_{SK2,3}(s) = \frac{(-P_2)(-P_3)}{(s-P_2)(s-P_3)}, \text{ it can be derived for } H_{SK2,3}.$$

$$R_1 = R_2 = 5\text{K}\Omega, \quad C_2 = 18.42\text{pF}, \quad C_1 = 21.55\text{pF}.$$



b) Chebyshev in SK.

$$n=3, \omega_0 = (2\pi)(1 \times 10^6), \epsilon = 0.1526$$

$$p_1 = -\omega_0(0.9694) \sin\left(-\frac{1}{6}\pi\right) + j\omega_0(1.3927) \cos\left(\frac{\pi}{6}\right) = -0.4847\omega_0 + j1.2061\omega_0$$

$$p_2 = -\omega_0(0.9694) \sin\left(\frac{3}{6}\pi\right) + j\omega_0(1.3927) \cos\left(\frac{3\pi}{6}\right) = -0.9496\omega_0$$

$$p_3 = -\omega_0(0.9694) \sin\left(\frac{5}{6}\pi\right) + j\omega_0(1.3927) \cos\left(\frac{5\pi}{6}\right) = -0.4847\omega_0 - j1.2061\omega_0$$

$$H_{SK}(s) = \frac{(-p_1)(-p_3)}{(s-p_1)(s-p_3)} = \frac{1.3^2 \omega_0^2}{s^2 + 0.9694\omega_0 s + (1.3)^2 \omega_0^2}$$

$$\omega_n = 1.3 \omega_0$$

$$\frac{\omega_n}{\alpha} = 0.9694 \omega_0 \Rightarrow \alpha = 1.3410.$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad \alpha = \frac{1}{R_1 + R_2} \sqrt{R_1 R_2 \frac{C_1}{C_2}}$$

$$\text{Let } R_1 = R_2 \Rightarrow \alpha = \frac{1}{2} \sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = (2\alpha)^2 = 7.1931.$$

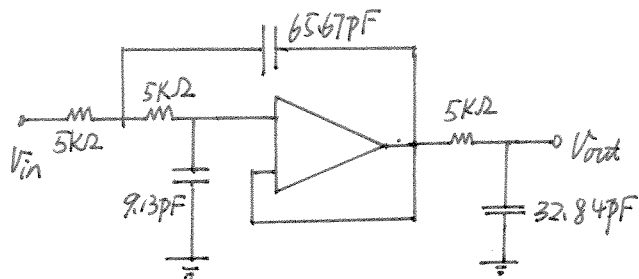
$$\omega_n = \frac{1}{\sqrt{7.1931} R_1 C_2} = (1.3)(2\pi)(1 \times 10^6)$$

$$\text{Let } R_1 = R_2 = 5 \text{ k}\Omega \Rightarrow C_1 = 9.13 \text{ pF} \Rightarrow C_1 = 65.67 \text{ pF}$$

$$p_2 = (2\pi)(0.9694 \times 10^6), \text{ and}$$

$$\frac{1}{R_3 C_3} = p_2$$

$$\text{Let } R_3 = 5 \text{ k}\Omega \Rightarrow C_3 = 32.84 \text{ pF}$$



c). Butterworth in SK.

$$n=3, \omega_0 = (2\pi)(1.27 \times 10^6)$$

$$P_k = \omega_0 \exp(j\frac{\pi}{2}) \exp(j\frac{2k-1}{2n}\pi), \quad k=1, 2, 3.$$

$$H_{p,3}(s) = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)} = \frac{[(2\pi)(1.27 \times 10^6)]^2}{s^2 - [4\pi \times (1.27 \times 10^6) \cos(\frac{2\pi}{3})]s + [2\pi(1.27 \times 10^6)]^2}$$

$$\omega_n = (2\pi)(1.27 \times 10^6)$$

$$\frac{\omega_n}{Q} = (4\pi)(1.27 \times 10^6) \cos(\frac{2\pi}{3}) \Rightarrow Q=1.$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad Q = \frac{1}{R_1 + R_2} \sqrt{R_1 R_2 \frac{C_1}{C_2}}.$$

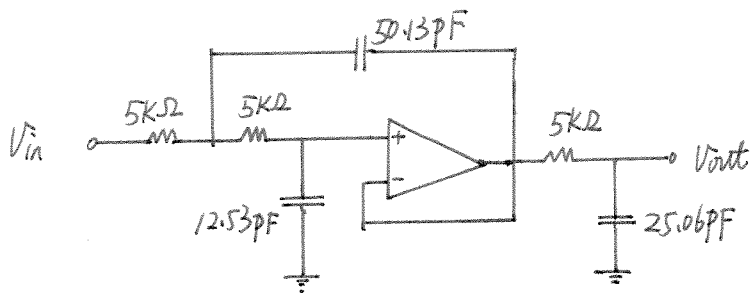
$$\text{Let } C_1 = 4C_2, R_1 = R_2$$

$$\omega_n = \frac{1}{2R_1 C_2} = (2\pi)(1.27 \times 10^6)$$

$$\text{Let } R_1 = R_2 = 5K\Omega \Rightarrow C_2 = 12.53 \text{ pF}, C_1 = 50.13 \text{ pF}.$$

$$P_2 = -\omega_0 = (2\pi)(1.27 \times 10^6) = \frac{1}{R_3 C_3}$$

$$\text{Let } R_3 = 5K\Omega \Rightarrow C_3 = 25.06 \text{ pF}$$



c). Chebyshev in SK.

$$n=3, \epsilon \pm 0.5089, \omega_0 = (2\pi)(10^6).$$

$$p_1 = -0.2470\omega_0 + j0.9660\omega_0$$

$$p_2 = -0.4941\omega_0$$

$$p_3 = -0.2470\omega_0 - j0.9660\omega_0$$

$$H_{p13}(s) = \frac{(-p_1)(-p_3)}{(s-p_1)(s-p_3)} = \frac{[(2\pi)(0.9971 \times 10^6)]^2}{s^2 + (0.4940)(2\pi)(10^6)s + (2\pi \times 0.9971 \times 10^6)^2}$$

$$\omega_n = (2\pi)(0.9971 \times 10^6)$$

$$\frac{\omega_n}{Q} = (0.4940)(2\pi \times 10^6) \Rightarrow Q = 2.02.$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad Q = \frac{1}{R_1 + R_2} \sqrt{R_1 R_2 \frac{C_1}{C_2}}.$$

$$\text{Let } R_1 = R_2 \Rightarrow Q = \frac{1}{2} \sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 16.296.$$

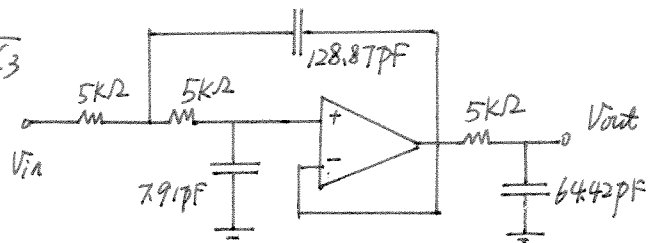
$$\omega_n = \frac{1}{\sqrt{16.296} R_1 C_2} = (2\pi)(0.9971 \times 10^6)$$

$$\text{Let } R_1 = R_2 = 5 \text{ k}\Omega \Rightarrow C_2 = 7.91 \text{ pF} \Rightarrow C_1 = 128.87 \text{ pF}$$

$$p_2 = 2\pi \times 0.4941 \times 10^6 = \frac{1}{R_3 C_3}$$

$$\text{Let } R_3 = 5 \text{ k}\Omega$$

$$\Rightarrow C_3 = 64.42 \text{ pF}$$



d). Butterworth in SK.

$$n=4, \quad \omega_0 = (2\pi)(1.3 \times 10^6)$$

$$P_k = \omega_0 \exp(j\frac{\pi}{2}) \exp(j\frac{2k-1}{2n}\pi), \quad k=1,2,3,4.$$

$$H_{SK,4}(s) = \frac{(-P_1)(-P_4)}{(s-P_1)(s-P_4)} = \frac{\omega_0^2}{s^2 - [2\omega_0 \cos(\frac{5\pi}{8})]s + \omega_0^2}$$

$$\omega_n = \omega_0 = (2\pi)(1.3 \times 10^6)$$

$$\frac{\omega_n}{a} = 4\pi(1.3 \times 10^6) \cos(\frac{5\pi}{8}) \Rightarrow Q = 1.31$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad Q = \frac{1}{R_1 + R_2} \sqrt{R_1 R_2 \frac{C_1}{C_2}}.$$

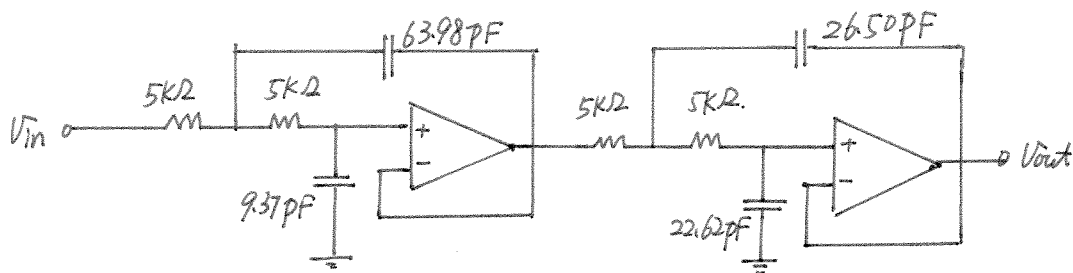
$$\text{Let } R_1 = R_2 \Rightarrow Q = \frac{1}{2} \sqrt{\frac{C_1}{C_2}} \Rightarrow \frac{C_1}{C_2} = 4Q^2 = 6.828$$

$$\omega_n = \frac{1}{\sqrt{6.828} R_1 C_2} = (2\pi)(1.3 \times 10^6)$$

$$\text{Let } R_1 = R_2 = 5k\Omega \Rightarrow C_2 = 9.37pF \Rightarrow C_1 = 63.98pF$$

Similarly, $H_{SK,3}(s) = \frac{(-P_2)(-P_3)}{(s-P_2)(s-P_3)}$. It can be derived that

$$R_1 = R_2 = 5k\Omega, \quad C_2 = 22.62pF, \quad C_1 = 26.50pF.$$



d). Chebyshev in SK.

$$n=3, \epsilon=0.3493, \omega_0=(2\pi)(1 \times 10^6)$$

$$P_1 = -\omega_0 0.6265 \sin\left(\frac{1}{6}\pi\right) + j\omega_0(1.1800) \cos\left(\frac{1}{6}\pi\right)$$

$$P_2 = -\omega_0 0.6265$$

$$P_3 = -\omega_0 0.6265 \sin\left(\frac{5}{6}\pi\right) + j\omega_0(1.1800) \cos\left(\frac{5}{6}\pi\right)$$

$$H_{p13} = \frac{(-P_1)(-P_3)}{(s-P_1)(s-P_3)} = \frac{[2\pi \times 1.069 \times 10^6]^2}{s^2 + (0.6265)(2\pi \times 10^6)s + (2\pi \times 1.069 \times 10^6)^2}$$

$$\omega_n = (2\pi)(1.069 \times 10^6)$$

$$\frac{\omega_n}{Q} = (0.6265)(2\pi \times 10^6) \Rightarrow Q = 1.7063$$

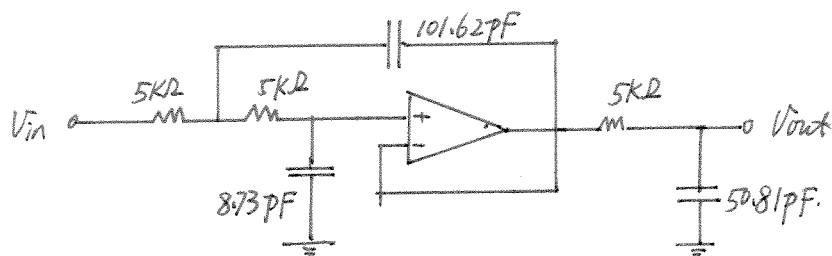
$$\text{Let } R_1 = R_2 \Rightarrow Q = \frac{1}{R_1 + R_2} \sqrt{R_1 R_2 \frac{C_1}{C_2}} = \frac{1}{2} \sqrt{\frac{Q}{C_2}} \Rightarrow \frac{Q}{C_2} = 4Q^2 = 11.6459.$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{\sqrt{11.6459} R_1 C_2} = (2\pi)(1.069 \times 10^6)$$

$$\text{Let } R_1 = R_2 = 5k\Omega \Rightarrow C_2 = 8.73pF \Rightarrow C_1 = 101.62pF.$$

$$-P_2 = (0.6265)(2\pi \times 10^6) = \frac{1}{R_3 C_3}$$

$$\text{Let } R_3 = 5k\Omega \Rightarrow C_3 = 50.81pF.$$



62) a). Butterworth TT

$$n=3, \omega_0 = (2\pi)(1.42 \times 10^6)$$

$$p_1 = \omega_0 \exp(j\frac{2\pi}{3}), p_2 = -\omega_0, p_3 = \omega_0 \exp(-j\frac{2\pi}{3})$$

$$H_{p,3} = \frac{[2\pi \times (1.42 \times 10^6)]^2}{s^2 - [4\pi \times (1.42 \times 10^6) \cos(\frac{2\pi}{3})]s + [2\pi \times 1.42 \times 10^6]^2}$$

$$\omega_n = (2\pi)(1.42 \times 10^6)$$

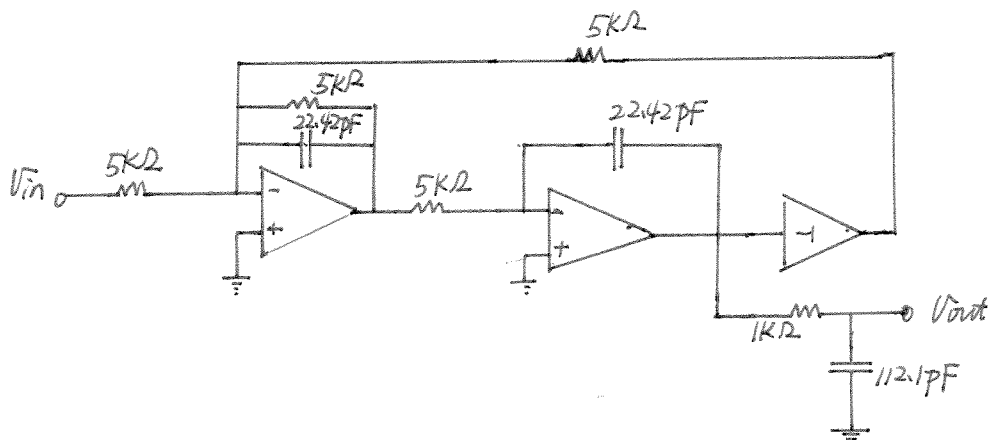
$$Q = \frac{-1}{2 \cos(\frac{2\pi}{3})} = 1$$

$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}, \quad Q = R_3 \sqrt{\frac{C_1}{C_2 R_2 R_4}}$$

$$\text{Let } R_2 = R_4 = R_3 = 5K\Omega, \quad C_1 = C_2 = 22.42 \text{ pF}$$

$$-P_2 = + (2\pi)(1.42 \times 10^6) = \frac{1}{R_3 C_3}$$

$$\text{Let } R_3 = 1K\Omega \Rightarrow C_3 = 112.1 \text{ pF}$$



$R_1 = R_2 = R_4$, to match low frequency gain requirement.

a) Chebyshev TT

$$n=2, \omega_0 = (2\pi)(1\text{MHz}), \epsilon = 0.3493$$

$$H_{p_{1,2}} = \frac{(1.2314)^2 \omega_0^2}{s^2 + 1.4256 \omega_0 s + (1.2314)^2 \omega_0^2}$$

$$\omega_n = (2\pi)(1.2314 \times 10^6)$$

$$Q = \frac{1.2314}{1.4256} = 0.8638$$

$$Q = R_3 \sqrt{\frac{C_1}{R_2 R_4 C_2}}, \quad \omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}$$

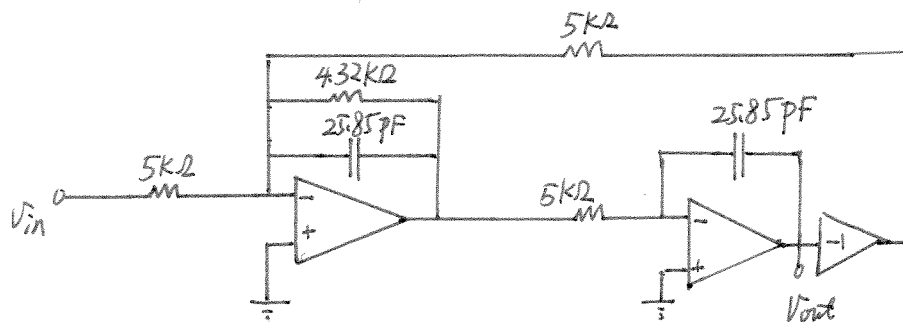
$$\text{Let } C_1 = C_2, R_2 = R_4$$

$$\omega_n = \frac{1}{R_2 C_1} = (2\pi)(1.2314 \times 10^6)$$

$$\text{Let } R_2 = R_4 = 5\text{k}\Omega, C_1 = C_2 = 25.85\text{pF}$$

$$R_1 = R_2 = R_4 = 5\text{k}\Omega, \text{ to match low frequency gain of unity.}$$

$$Q = \frac{R_3}{R_2} \Rightarrow R_3 = 4.32\text{k}\Omega.$$



b). Butterworth with TT

$$n=4, \omega_0 = (2\pi)(1.6 \times 10^6)$$

$$P_1 = \omega_0 \exp(j\frac{5\pi}{8}), \quad P_4 = \omega_0 \exp(-j\frac{5\pi}{8})$$

$$P_2 = \omega_0 \exp(j\frac{7\pi}{8}), \quad P_3 = \omega_0 \exp(-j\frac{7\pi}{8})$$

$$H_{p_{1,4}} = \frac{\omega_0^2}{s^2 - [2\omega_0 \cos(\frac{5\pi}{8})]s + \omega_0^2}$$

$$\omega_n = \omega_0 = (2\pi)(1.6 \times 10^6)$$

$$\frac{\omega_n}{Q} = (4\pi)(1.6 \times 10^6) \cos(\frac{5\pi}{8}) \Rightarrow Q = 1.31$$

$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}, \quad Q = R_3 \sqrt{\frac{C_1}{R_2 R_4 C_2}}$$

$$\text{Let } R_2 = R_4, \quad C_1 = C_2 \Rightarrow \omega_n = \frac{1}{R_2 C_1} = (2\pi)(1.6 \times 10^6)$$

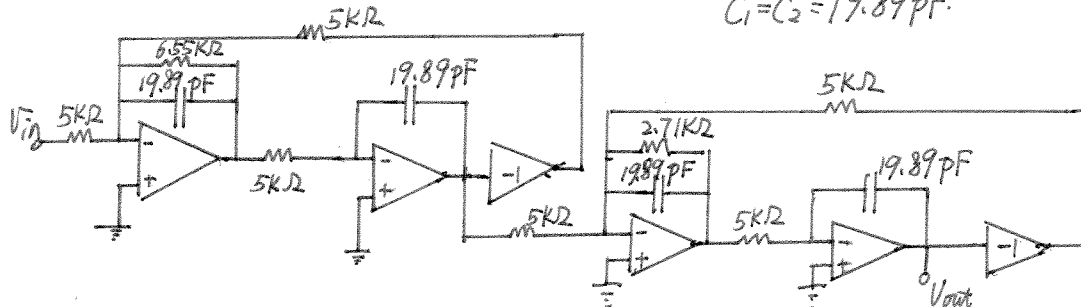
$$\text{Let } R_2 = R_4 = 5K\Omega \Rightarrow C_1 = C_2 = 19.89 \text{ pF.}$$

$R_1 = R_2 = R_4 = 5K\Omega$, to obtain a low-frequency gain of unity.

$$Q = \frac{R_3}{R_2} = 1.31 \Rightarrow R_3 = 1.31 R_2 = 6.55 K\Omega.$$

Similarly for $H_{p_{2,3}}$, it can be derived that $R_1 = R_2 = R_4 = 5K\Omega, R_3 = 2.7 K\Omega$

$$C_1 = C_2 = 19.89 \text{ pF.}$$



b). Chebyshev with TT

$$n=3, \epsilon = 0.1526, \omega_0 = (2\pi)(1 \times 10^6)$$

$$P_{1,3} = 0.4847\omega_0 \pm j 1.2061\omega_0$$

$$P_2 = -0.9694\omega_0$$

$$H_{p,3}(s) = \frac{(1.3)^2 \omega_0^2}{s^2 + 0.9694\omega_0 s + (1.3)^2 \omega_0^2}$$

$$\omega_n = 1.3\omega_0, \quad \frac{\omega_n}{Q} = 0.9694\omega_0$$

$$Q = \frac{1.3}{0.9694} = 1.3410$$

$$Q = R_3 \sqrt{\frac{C_1}{R_2 R_4 C_2}}, \quad \omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}$$

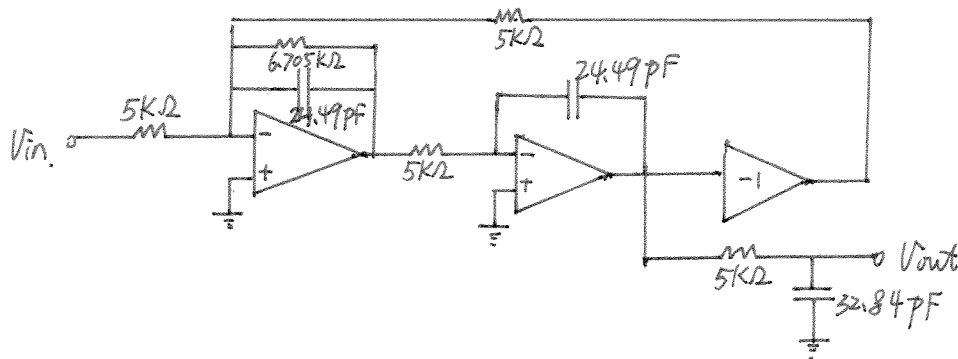
$$\text{Let } R_2 = R_4, C_1 = C_2, \quad \frac{1}{R_2 C_1} = (1.3)(2\pi)(10^6)$$

$$\text{Let } R_2 = R_4 = 5K\Omega, \Rightarrow C_1 = C_2 = 24.49 \text{ pF}$$

$$R_1 = R_2 = R_4 = 5K\Omega, \text{ to obtain low-frequency gain of unity.}$$

$$R_3 = Q R_2 = 6.705K\Omega$$

$$-P_2 = (2\pi)(0.9694 \times 10^6) = \frac{1}{R_5 C_5}, \text{ Let } R_5 = 5K\Omega \Rightarrow C_5 = 32.84 \text{ pF}$$



c) Butterworth with TT

$$n=3, \omega_0 = (2\pi)(1.27 \times 10^6)$$

$$P_1 = \omega_0 \exp(j\frac{2\pi}{3}), \quad P_3 = \omega_0 \exp(-j\frac{2\pi}{3}), \quad P_2 = -\omega_0.$$

$$H_{p13}(s) = \frac{[(2\pi)(1.27 \times 10^6)]^2}{s^2 - [(4\pi)(1.27 \times 10^6) \cos(\frac{2\pi}{3})]s + [(2\pi)(1.27 \times 10^6)]^2}$$

$$\omega_n = (2\pi)(1.27 \times 10^6), \quad \frac{\omega_n}{Q} = (4\pi)(1.27 \times 10^6) \cos(\frac{2\pi}{3})$$

$$Q = -\frac{1}{2 \cos(\frac{2\pi}{3})} = 1.$$

$$Q = R_3 \sqrt{\frac{C_1}{R_2 R_4 C_2}}, \quad \omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}$$

$$\text{Let } R_2 = R_4, \quad C_1 = C_2, \quad \frac{1}{R_2 C_1} = (2\pi)(1.27 \times 10^6)$$

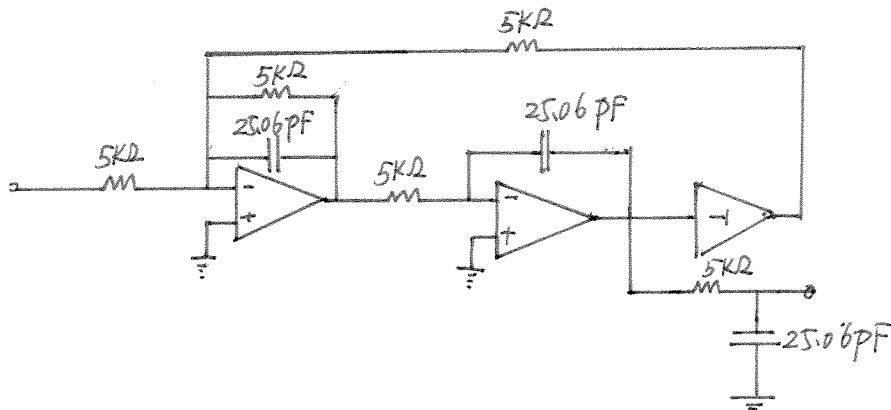
$$\text{Let } R_2 = R_4 = 5K\Omega \Rightarrow C_1 = C_2 = 25.06 \text{ pF}.$$

$$R_1 = R_2 = R_4 = 5K\Omega, \text{ to obtain Low-frequency gain of unity.}$$

$$R_3 = Q R_2 = 5K\Omega.$$

$$-P_2 = \omega_0 \Rightarrow \frac{1}{R_5 C_5} = (2\pi)(1.27 \times 10^6)$$

$$\text{Let } R_5 = 5K\Omega \Rightarrow C_5 = 25.06 \text{ pF}$$



c) Chebyshev TT

$$n=3, \epsilon=0.5089, \omega_0 = (2\pi)(1 \times 10^6)$$

$$P_{1,3} = -0.2470 \omega_0 \pm j 0.9660 \omega_0, \quad P_2 = -0.4941 \omega_0$$

$$H_{P_{1,3}}(s) = \frac{[(2\pi)(0.9971 \times 10^6)]^2}{s^2 + (0.4940)(2\pi \times 10^6)s + [2\pi \times 0.9771 \times 10^6]^2}$$

$$\omega_n = (2\pi)(0.9971 \times 10^6)$$

$$\frac{\omega_n}{Q} = (0.4940)(2\pi \times 10^6) \Rightarrow Q = 2.02$$

$$Q = R_3 \sqrt{\frac{C_1}{R_2 R_4 C_2}}, \quad \omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}$$

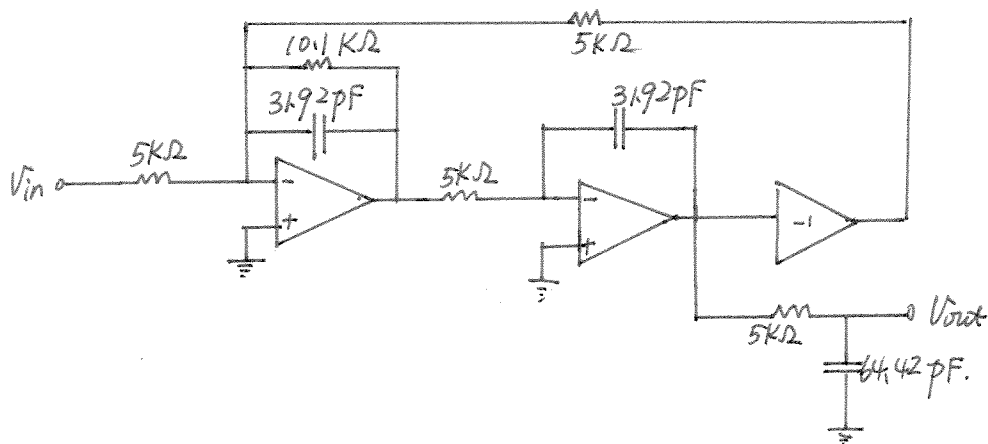
$$\text{Let } R_2 = R_4, C_1 = C_2 \Rightarrow \frac{1}{R_2 C_1} = (2\pi)(0.9971 \times 10^6)$$

$$\text{Let } R_2 = R_4 = 5 \text{ k}\Omega \Rightarrow C_1 = C_2 = 31.92 \text{ pF}$$

$R_1 = R_2 = R_4 = 5 \text{ k}\Omega$, to obtain Low-frequency gain of unity.

$$R_3 = Q R_2 = 10.1 \text{ k}\Omega.$$

$$-P_2 = (2\pi)(0.4941 \times 10^6) = \frac{1}{R_5 C_5}. \quad \text{Let } R_5 = 5 \text{ k}\Omega \Rightarrow C_5 = 64.42 \text{ pF}.$$



d). Butterworth in TT

$$n=4, \omega_0 = (2\pi)(1.3 \times 10^6)$$

$$P_{1,4} = \omega_0 \exp(\pm j \frac{5\pi}{8}), \quad P_{2,3} = \omega_0 \exp(\pm j \frac{7\pi}{8})$$

$$H_{P_{1,4}}(s) = \frac{\omega_0^2}{s^2 - [2\omega_0 \cos(\frac{5\pi}{8})]s + \omega_0^2}$$

$$\omega_n = (2\pi)(1.3 \times 10^6)$$

$$\frac{\omega_n}{Q} = (4\pi)(1.3 \times 10^6) \cos(\frac{5\pi}{8}) \Rightarrow Q = -\frac{1}{2\cos(\frac{5\pi}{8})} = 1.31.$$

$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}, \quad Q = R_3 \sqrt{\frac{C_1}{R_2 R_4 C_2}}$$

$$\text{Let } R_2 = R_4, C_1 = C_2, \quad \omega_n = \frac{1}{\sqrt{R_2^2 C_1^2}} = \frac{1}{R_2 C_1} = (2\pi)(1.3 \times 10^6)$$

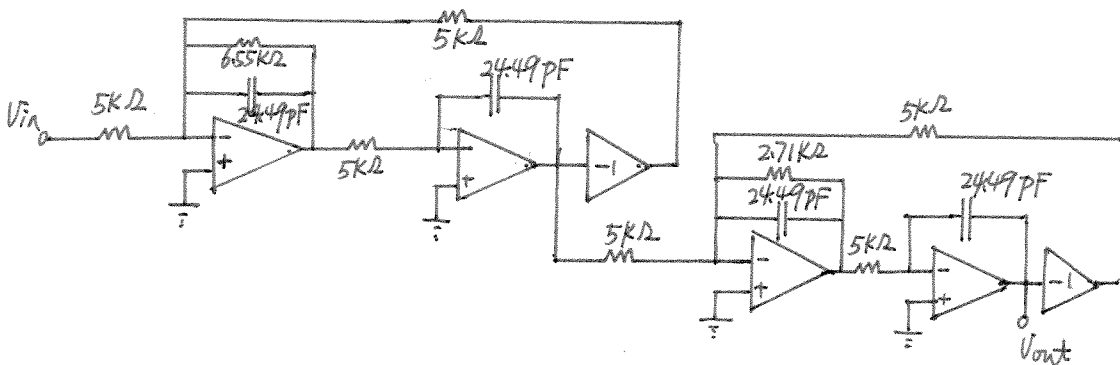
$$\text{Let } R_2 = R_4 = 5k\Omega \Rightarrow C_1 = C_2 = 24.49 \text{ pF}.$$

$$R_1 = R_2 = R_4 = 5k\Omega, \text{ to obtain a low-frequency gain of unity.}$$

$$R_3 = Q R_2 = 6.55k\Omega.$$

$$\text{Similarly, } H_{P_{2,3}}(s) = \frac{\omega_0^2}{s^2 - [2\omega_0 \cos(\frac{7\pi}{8})]s + \omega_0^2}$$

It can be obtained that $R_1 = R_2 = R_4 = 5k\Omega$, $R_3 = 2.71k\Omega$, $C_1 = C_2 = 24.49 \text{ pF}$.



d). Chebyshev TT

$$n=3, \epsilon = 0.3493, \omega_0 = (2\pi)(1 \times 10^6)$$

$$P_{1,3} = -0.3133 \omega_0 \pm j 1.022 \omega_0, \quad P_2 = -0.6265 \omega_0$$

$$H_{p,3} = \frac{[2\pi \times 1.069 \times 10^6]^2}{s^2 + (0.6265)(2\pi \times 10^6)s + (2\pi \times 1.069 \times 10^6)^2}$$

$$\omega_n = (2\pi)(1.069 \times 10^6)$$

$$\frac{\omega_n}{Q} = (0.6265)(2\pi \times 10^6) \Rightarrow Q = \frac{1.069}{0.6265} = 1.7063$$

$$\omega_n = \frac{1}{\sqrt{R_2 R_4 C_1 C_2}}, \quad Q = R_3 \sqrt{\frac{C_1}{R_2 R_4 C_2}}$$

$$\text{Let } R_2 = R_4, \quad C_1 = C_2, \quad \frac{1}{R_2 C_1} = (2\pi)(1.069 \times 10^6)$$

$$\text{Let } R_2 = R_4 = 5 \text{ k}\Omega \Rightarrow C_1 = C_2 = 29.78 \text{ pF}$$

$R_1 = R_2 = R_4 = 5 \text{ k}\Omega$, to obtain a low-frequency gain of unity.

$$R_3 = Q R_2 = 8.53 \text{ k}\Omega$$

$$-P_2 = 0.6265 \times (2\pi \times 10^6) = \frac{1}{R_5 C_5}, \quad \text{Let } R_5 = 5 \text{ k}\Omega \Rightarrow C_5 = 50.81 \text{ pF}$$

